



GEOMETRY →

Geometry For Middle School Teachers

**A Professional Development Program
to Implement the
2001 Virginia Standards of Learning**

March 2004

**Office of Middle Instructional Services
Virginia Department of Education
P.O. Box 2120
Richmond, Virginia 23218-2120**



Copyright © 2004 by the

Virginia Department of Education
P.O. Box 2120
Richmond, Virginia 23218-2120
www.pen.k12.va.us

All rights reserved. Reproduction of these materials for instructional purposes in Virginia classrooms is permitted.

Superintendent of Public Instruction

Jo Lynne DeMary

Assistant Superintendent for Instruction

Patricia I. Wright

Office of Middle Instructional Services

James Firebaugh, Director

Lois Williams, Middle Mathematics Specialist

Office of Elementary Instructional Services

Karen Grass, Elementary Mathematics Specialist

Notice to Reader

In accordance with the requirements of the Civil Rights Act and other federal and state laws and regulations, this document has been reviewed to ensure that it does not reflect stereotypes based on sex, race, or national origin.

The Virginia Department of Education does not unlawfully discriminate on the basis of sex, race, age, color, religion, handicapping conditions, or national origin in employment or in its educational programs and activities.

The activity that is the subject of this report was supported in whole or in part by the U.S. Department of Education. However, the opinions expressed herein do not necessarily reflect the position or policy of the U.S. Department of Education, and no official endorsement by the U.S. Department of Education should be inferred.



Acknowledgements

The Virginia Department of Education wishes to express sincere appreciation to the following individuals who have contributed to the writing and editing of the activities in this document.

**Margie Mason, Associate Professor
The College of William and Mary
Williamsburg, Virginia**

**Bruce Mason, Consultant
Williamsburg, Virginia**

**Dana Johnson, Instructor
The College of William and Mary
Williamsburg, Virginia**

**Carol L. Rezba, Assistant Professor
Longwood University**

**Christa Southall
Stafford County**



Introduction

The updated *Geometry for Middle School Teachers* is a professional development program designed to assist teachers in implementing the 2001 Virginia Mathematics Standards of Learning. This professional development program provides a sample of meaningful and engaging activities correlated to the geometry strand of the grades 6-8 Mathematics Standards of Learning.

The purpose of the professional development program is to enhance teachers' content knowledge and their use of instructional strategies for teaching the geometry strand of the Mathematics Standards of Learning. Teachers will learn about the van Hiele model for the development of geometric thought and how this can be used to guide instruction and classroom assessment. Through explorations, problem solving, and hands-on experiences, teachers will engage in discussions and activities that address many of the dimensions of geometry including spatial relationships, properties of geometric figures and solids, constructions, geometric modeling, geometric transformations, coordinate geometry, the geometry of measurement, informal geometric reasoning, and geometric connections to the physical world. Teachers will explore two- and three-dimensional figures, paper folding, tessellations, and the use of other manipulatives and technology to develop geometric understanding. Through these activities, it is anticipated that teachers will develop new techniques that assist in increasing student achievement in their classrooms.

Designed to be presented by teacher trainers, this professional development program includes directions for the trainer, as well as the activity sheets for participants. An addendum to the module includes video segments of the van Hiele levels. These video segments portray students engaged in assessment tasks and can be used to discuss students' level of development of geometric thought. Directions for the assessment tasks are also included.

Trainers should adapt the materials to best fit the needs of their audience; adding materials that may be more appropriate for their audience and eliminating materials that have been used in previous training sessions. All materials in this document may be duplicated and distributed as desired for use in public schools in Virginia.

The training programs are organized into five modules that may be offered by school divisions for teacher licensure renewal points or for a one-credit graduate course, when university credit can be arranged.



GLOSSARY

<i>Acute Angle</i>	An angle with a measure greater than 0 degrees but less than 90 degrees.
<i>Acute Triangle</i>	A triangle with three acute angles (or no angle measuring 90 degrees or greater).
<i>Adjacent Sides</i>	Two sides of a polygon with a common vertex.
<i>Angle</i>	Two rays that share an endpoint.
<i>Arc</i>	Part of a circle.
<i>Area</i>	The amount of surface in a region or enclosed within a boundary. Area is measured in square units such as square feet or square centimeters.
<i>Attribute</i>	A characteristic possessed by an object. Characteristics include shape, color, size, length, weight, capacity, area, etc.
<i>Base of a Solid</i>	A plane figure. If the solid is a cylinder or prism, there are two bases that are parallel and congruent.
<i>Centimeter</i>	A metric unit of length equal to one-hundredth of one meter.
<i>Circle</i>	A closed curve with all points in one plane and equidistant from a fixed point (the center).
<i>Circumference</i>	The length of the boundary of a circular region. The circumference can be computed by multiplying the diameter by π (π), a number a little more than 3.14.
<i>Concentric Circles</i>	Two or more circles that have the same center and different radii.
<i>Cone</i>	A three-dimensional figure with one curved surface, one flat surface (usually circular), one curved edge, and one vertex.
<i>Congruent</i>	Having exactly the same size and shape. Congruent polygons have their corresponding angles congruent and corresponding sides congruent.
<i>Coordinate System</i>	A reference system for locating and graphing points. In two dimensions, a coordinate system usually consists of a horizontal axis and a vertical axis, which intersect at the origin. Each point in the plane is located by its horizontal distance and vertical distance from the origin. These distances, or coordinates, form an ordered pair of numbers.
<i>Cube</i>	A solid figure in which every face is a square and every edge is the same length.



GEOMETRY

<i>Cubic Foot</i>	The volume of a cube that is one foot wide, one foot high, and one foot long.
<i>Cubic Unit</i>	A unit of measure that has a length of one unit, a width of one unit, and a height of one unit used to measure volume. Examples are cubic inches, cubic centimeters, etc.
<i>Cylinder</i>	A solid figure formed by two congruent parallel circles joined by a curved surface.
<i>Decagon</i>	A polygon with ten sides. A regular decagon has ten congruent sides and ten congruent angles.
<i>Diagonal</i>	A line segment that joins two non-adjacent vertices of a polygon or polyhedron.
<i>Diameter</i>	A line segment passing through the center of a circle or sphere and connecting two points on the circle or sphere.
<i>Diamond</i>	See <i>Rhombus</i> .
<i>Dimension</i>	The number of coordinates used to express a position.
<i>Dodecagon</i>	A polygon with twelve sides. A regular dodecagon has twelve congruent sides and twelve congruent angles.
<i>Dodecahedron</i>	A polyhedron with twelve faces. All faces of a regular dodecahedron are congruent, regular pentagons.
<i>Edge</i>	A line segment where two faces of a three-dimensional figure intersect.
<i>Endpoint</i>	The point(s) at the end of a ray or line segment.
<i>Equilateral Triangle</i>	A triangle with three congruent sides. Each angle measures 60 degrees.
<i>Face</i>	A plane figure that serves as one side of a solid figure.
<i>Flip</i>	See <i>Reflection</i> .
<i>Geometry</i>	The branch of mathematics that deals with the position, size, and shape of figures.
<i>Grid</i>	A network of horizontal and vertical lines that intersect to form squares or rectangles.
<i>Hemisphere</i>	Half of a sphere, formed by making a plane cut through the center of a sphere.



<i>Heptagon</i>	A polygon with seven sides. A regular heptagon has seven congruent sides and seven congruent angles.
<i>Hexagon</i>	A polygon with six sides. A regular hexagon has six congruent sides and six congruent angles.
<i>Hexahedron</i>	A polyhedron with six faces. All faces of a regular hexahedron are congruent squares. A regular hexahedron is a cube.
<i>Hypotenuse</i>	The side opposite the right angle of a right triangle. The hypotenuse is the longest side of a right triangle.
<i>Icosahedron</i>	A polyhedron with twenty faces. All faces of a regular icosahedron are congruent, equilateral triangles.
<i>Isosceles Triangle</i>	A triangle with at least two congruent sides and two congruent angles. An equilateral triangle is a special case of an isosceles triangle.
<i>Kite</i>	A convex quadrilateral with two distinct pairs of adjacent, congruent sides.
<i>Line</i>	A set of points that forms a straight path extending infinitely in two directions. Lines are often called “straight lines” to distinguish them from curves, which are often called “curved lines.” Part of a line with two endpoints is called a “line segment.”
<i>Line of Symmetry</i>	A line dividing a two-dimensional figure into two parts that are mirror images of each other.
<i>Line Segment</i>	A part of a line. A line segment has two endpoints and a finite length.
<i>Network</i>	A diagram consisting of arcs (branches) connecting points or nodes (junctions). A network may represent a real-world situation, such as a road system or an electronic circuit. Sometimes the nodes are called vertices.
<i>Node</i>	A point in a network at the end of an arc or at the junction of two or more arcs.
<i>Nonagon</i>	A polygon with nine sides. A regular nonagon has nine congruent sides and nine congruent angles.
<i>Obtuse Angle</i>	An angle that is greater than 90 degrees but less than 180 degrees; that is, between a right angle and a straight line.
<i>Obtuse Triangle</i>	A triangle that has one obtuse angle.
<i>Octagon</i>	A polygon with eight sides. A regular octagon has eight congruent sides and eight congruent angles.



<i>Octahedron</i>	A polyhedron with eight faces. All faces of a regular octahedron are congruent, equilateral triangles.
<i>Opposite Angles</i>	In a quadrilateral, angles that do not have a common side; non-adjacent angles.
<i>Parallel Lines</i>	Lines lying in the same plane that are always the same distance apart.
<i>Parallelogram</i>	A quadrilateral with both pairs of opposite sides parallel. Opposite angles are congruent.
<i>Pentagon</i>	A polygon with five sides. A regular pentagon has five congruent sides and five congruent angles.
<i>Perimeter</i>	The distance around a figure.
<i>Perpendicular</i>	At right angles.
<i>pi (π)</i>	The ratio of the circumference of a circle to its diameter. This ratio is the same for every circle. Its value, which is found by dividing the circumference by the diameter, is a little more than 3.14.
<i>Pie graph</i>	A circle marked into sectors. Each sector shows the fraction represented by one category of data. Pie graphs are also called circle graphs.
<i>Plane</i>	A flat surface extending infinitely in all directions.
<i>Plane Figure</i>	In geometry, a closed two-dimensional figure that lies entirely in one plane. (Polygons and circles are examples of plane figures. An arc is not a plane figure because it is not closed.)
<i>Point</i>	The smallest geometric unit. A position in space, often represented by a dot.
<i>Polygon</i>	A simple, closed, plane figure formed by straight sides.
<i>Polyhedron</i>	A three-dimensional figure in which all the surfaces are polygons.
<i>Prism</i>	A polyhedron with at least one pair of opposite faces that are parallel and congruent. Corresponding edges of these faces are joined by rectangles or parallelograms.
<i>Pyramid</i>	A polyhedron with any polygon for its base. The other faces are triangles that meet at a point or vertex.
<i>Quadrilateral</i>	A polygon with four sides.



<i>Ray</i>	A set of points that form a straight path extending infinitely in one direction. A ray has one endpoint.
<i>Rectangle</i>	A parallelogram with four right angles. Opposite sides are congruent and parallel.
<i>(Right) Rectangular Prism</i>	A solid figure in which all six faces are rectangles. Opposite faces are parallel and congruent.
<i>Reflection</i>	A transformation of a geometric figure that results in a mirror image of the original.
<i>Regular Polygon</i>	A polygon that has congruent sides and congruent angles.
<i>Regular Polyhedron</i>	A polyhedron with congruent faces that are regular polygons.
<i>Rhombus</i>	A parallelogram with four congruent sides. Opposite angles are congruent.
<i>Right Angle</i>	An angle that is one-fourth of a full turn. A right angle measures 90 degrees.
<i>Right Triangle</i>	A triangle that has one right angle.
<i>Scalene Triangle</i>	A triangle with no sides congruent.
<i>Semicircle</i>	An arc that is one-half of a circle.
<i>Similar</i>	Figures that have the same shape but not necessarily the same size. Similar polygons have corresponding angles congruent and corresponding sides in proportion. Congruent is a special case of similar where the ratio of the corresponding sides is 1-1.
<i>Slide</i>	See <i>Translation</i>
<i>Solid Figure</i>	A closed, three-dimensional figure.
<i>Sphere</i>	A three-dimensional figure formed by a set of points that are all the same distance from a fixed point called the center.
<i>Square</i>	A rectangle with congruent sides.
<i>Square Unit</i>	A unit of measure that has a length of one unit and a width of one unit used to measure area. Examples are square inches, square centimeters, acres, etc.
<i>Surface</i>	Part or all of the boundary of a solid. A surface may be flat or curved. (For example, a cone has one flat surface and one curved surface.)



<i>Symmetry</i>	<ul style="list-style-type: none">a. If a figure can be folded along a line so that the two halves match exactly, then the figure has line symmetry.b. If a figure can be turned less than 360 degrees about a point and fit exactly on itself, then a figure has rotational symmetry.
<i>Tessellation</i>	An arrangement of plane figures (usually congruent figures) to cover a surface without overlapping or leaving any gaps.
<i>Tetrahedron</i>	A polyhedron with four triangular faces. A tetrahedron is a triangular pyramid.
<i>Three-Dimensional</i>	Relating to objects that have length, width, and depth. Solid figures such as polyhedra, cones, and spheres are three-dimensional.
<i>Transformation</i>	Changing a geometric figure, according to a rule. Examples of transformations are reflection, rotation, and translation.
<i>Translation</i>	A transformation in which a geometric figure is formed by moving every point on a figure the same distance in the same direction.
<i>Trapezoid</i>	A quadrilateral with exactly one pair of parallel sides.
<i>Triangle</i>	A polygon with three sides.
<i>Triangular Prism</i>	A prism in which the bases are triangles.
<i>Two-Dimensional</i>	Relating to figures that have length and width but not depth. Figures such as polygons and circles are two-dimensional.
<i>Vertex</i>	<ul style="list-style-type: none">a. A point at which two line segments, lines, or rays meet to form an angle.b. A point on a polyhedron where three or more faces intersect.c. The point opposite the base of a cone or pyramid. <p>The plural of vertex is vertices.</p>



Table of Contents

	PAGE
Session I	
van Hiele Theory of Geometric Thought	3
van Hiele Levels of Geometric Thought	4
Quadrilaterals and their Properties	18
Quadrilateral Sort	19
What's My Rule?	22
Quadrilateral Properties Laboratory	24
Quadrilateral Sorting Laboratory	28
Plane Figures and their Properties	32
Polygons and the Geoboard	33
A Country Mile	35
Similarity and Congruence with Geostrips	36
Perimeters and Areas of Similar Triangles	38
Making and Using a Hypsometer	41
Human Circle	43
Session II	
Spatial Relationships	47
Square It	48
Pick Up the Toothpicks	50
Partition the Square	52
Tangrams	54
Make Your Own Tangrams	55
Area and Perimeter Problems with Tangrams	57
Spatial Problem Solving with Tangrams	60
Spatial Relations Using the Soma Cube	63
Constructing the Soma Pieces	64
Building the Soma Cube	69
Making 2-Dimensional Drawings of 3-Dimensional Figures	73
Cube Structures	75



Session III

Transformational Geometry: Tessellations	78
Sum of the Measures of Angles of a Triangle	80
Do Congruent Triangles Tessellate?	83
Do Congruent Quadrilaterals Tessellate?	90
Tessellations by Translation	95
Tessellations by Rotation	98

Session IV

Circles	101
Circumference vs. Diameter	102
Cake Problem	103
Problem Solving with Circles	105
Pythagorean Theorem	110
Geoboard Exploration of the Pythagorean Relationship	111
Egyptian Rope Stretching	113
Solid Geometry	114
Polyhedron Sort	115
What's My Shape? Ask Me About It.	116
What's My Shape? Touch Me.	117
Creating Nets	118
Building Polyhedra	120
6 Flexagon	121
Cover It Up	123
Architect's Square	126

Session V

Perimeter and Area, Part I	131
Dominoes and Triominoes	132
Tetrominoes	135
Pentominoes	138
Areas with Pentominoes	142
Hexominoes	149
Perimeters with Hexominoes	153
Perimeter and Area, Part II	155
The Perimeter Is 24 Inches. What Is The Area?	156
The Area Is Square 24 Inches. What Is The Perimeter?	159
Change the Area	160



Geometry Standards of Learning Kindergarten through Eighth Grade

* indicates a related Standard of Learning from the
Measurement Strand

- K.11 The student will identify, describe, and draw two-dimensional (plane) geometric figures (circle, triangle, square, and rectangle).
- K.12 The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their position and orientation in space.
- K.13 The student will compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).
- 1.15 The student will describe the proximity of objects in space (*near, far, close by, below, above, up, down, beside, and next to*).
- 1.16 The student will draw, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, corners, and square corners.
- 1.17 The student will identify and describe objects in his/her environment that depict plane geometric figures (triangle, rectangle, square, and circle).
- 2.20 The student will identify, describe, and sort three-dimensional (solid) concrete figures, including a cube, rectangular solid (prism), square pyramid, sphere, cylinder, and cone, according to the number and shape of the solid's faces, edges, and corners.
- 2.21 The student will identify and create figures, symmetric along a line, using various concrete materials.
- 2.22 The student will compare and contrast plane and solid geometric shapes (circle/sphere, square/cube, and rectangle/rectangular solid).
- 3.18 The student will analyze two-dimensional (plane) and three-dimensional (solid) geometric figures (circle, square, rectangle, triangle, cube, rectangular solid [prism], square pyramid, sphere, cone, and cylinder) and identify relevant properties, including the number of corners, square corners, edges, and the number and shape of faces, using concrete models.



- 3.19 The student will identify and draw representations of line segments and angles, using a ruler or straightedge.
- 3.20 The student, given appropriate drawings or models, will identify and describe congruent and symmetrical, two-dimensional (plane) figures, using tracing procedures.
- 4.13* The student will
- a) identify and describe situations representing the use of perimeter and area; and
 - b) use measuring devices to find perimeter in both standard and nonstandard units of measure. (*Measurement strand)
- 4.14 The student will investigate and describe the relationships between and among points, lines, line segments, and rays.
- 4.15 The student will
- a) identify and draw representations of points, lines, line segments, rays, and angles, using a straightedge or ruler; and
 - b) describe the path of shortest distance between two points on a flat surface.
- 4.16 The student will identify and draw representations of lines that illustrate intersection, parallelism, and perpendicularity.
- 4.17 The student will
- a) analyze and compare the properties of two-dimensional (plane) geometric figures (circle, square, rectangle, triangle, parallelogram, and rhombus) and three-dimensional (solid) geometric figures (sphere, cube, and rectangular solid [prism]);
 - b) identify congruent and noncongruent shapes; and
 - c) investigate congruence of plane figures after geometric transformations such as reflection (flip), translation (slide) and rotation (turn), using mirrors, paper folding, and tracing.
- 4.18 The student will identify the ordered pair for a point and locate the point for an ordered pair in the first quadrant of a coordinate plane.
- 5.8* The student will describe and determine the perimeter of a polygon and the area of a square, rectangle, and right triangle, given the appropriate measures. (*Measurement strand)



- 5.10* The student will differentiate between perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation. (*Measurement strand)
- 5.13* The student will measure and draw right, acute, and obtuse angles and triangles, using appropriate tools. (*Measurement strand)
- 5.14 The student will classify angles and triangles as right, acute, or obtuse.
- 5.15 The student, using two-dimensional (plane) figures (square, rectangle, triangle, parallelogram, rhombus, kite, and trapezoid) will
- a) recognize, identify, describe, and analyze their properties in order to develop definitions of these figures;
 - b) identify and explore congruent, noncongruent, and similar figures;
 - c) investigate and describe the results of combining and subdividing shapes;
 - d) identify and describe a line of symmetry; and
 - e) recognize the images of figures resulting from geometric transformations such as translation (slide), reflection (flip), or rotation (turn).
- 5.16 The student will identify, compare, and analyze properties of three-dimensional (solid) geometric shapes (cylinder, cone, cube, square pyramid, and rectangular prism).

Grade 6

- 6.11* The student will determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area and apply the appropriate formula. (*Measurement strand)
- 6.12* The student will
- a) solve problems involving the circumference and/or area of a circle when given the diameter or radius: and
 - b) derive approximations for pi (B) from measurements for circumference and diameter, using concrete materials or computer models. (*Measurement strand)
- 6.13* The student will
- a) estimate angle measures, using 45° , 90° , and 180° , referents, and use the appropriate tools to measure the given angles; and
 - b) measure and draw right, acute, and obtuse angles and triangles. (*Measurement strand)
- 6.14 The student will identify, classify, and describe the characteristics of plane figures, describing their similarities, differences, and defining properties.



- 6.15 The student will determine congruence of segments, angles, and polygons by direct comparison, given their attributes. Examples of noncongruent and congruent figures will be included.
- 6.16 The student will construct the perpendicular bisector of a line segment and an angle bisector.
- 6.17 The student will sketch, construct models of, and classify solid figures (rectangular prism, cone, cylinder, and pyramid).

Grade 7

- 7.7 The student, given appropriate dimensions, will
- a) estimate and find the area of polygons by subdividing them into rectangles and right triangles; and
 - b) apply perimeter and area formulas in practical situations. (*Measurement strand)
- 7.8 The student will investigate and solve problems involving the volume and surface area of rectangular prisms and cylinders, using concrete materials and practical situations to develop formulas. (*Measurement strand)
- 7.9 The student will compare and contrast the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. Deductive reasoning and inference will be used to classify quadrilaterals.
- 7.10 The student will identify and draw the following polygons: pentagon, hexagon, heptagon, octagon, nonagon, and decagon.
- 7.11 The student will determine if geometric figures — quadrilaterals and triangles — are similar and write proportions to express the relationships between corresponding parts of similar figures.
- 7.12 The student will identify and graph ordered pairs in the four quadrants of a coordinate plane.
- 7.13 The student, given a polygon in the coordinate plane, will represent transformations — rotation and translation — by graphing the coordinates of the vertices of the transformed polygon and sketching the resulting figure.



Grade 8

- 8.6* The student will verify by measuring and describe the relationships among vertical angles, supplementary angles, and complementary angles and will measure and draw angles of less than 360° .(*Measurement strand)
- 8.7* The student will investigate and solve practical problems involving volume and surface area of rectangular solids (prisms), cylinders, cones, and pyramids. (*Measurement strand)
- 8.8 The student will apply transformations (rotate or turn, reflect or flip, translate or slide, and dilate or scale) to geometric figures represented on graph paper. The student will identify applications of transformations, such as tiling, fabric design, art, and scaling.
- 8.9 The student will construct a three-dimensional model, given the top, side, and/or bottom views.
- 8.10 The student will
- verify the Pythagorean Theorem, using diagrams, concrete materials, and measurement; and
 - apply the Pythagorean Theorem to find the missing length of a side of a right triangle when given the lengths of the other two sides.



Middle School Geometry Session 1

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
van Hiele Theory of Geometric Thought	Lecture – van Hiele Levels of Geometric Thought Triangle Sorts	4	6.14, 6.15	Explanation Sheets -van Hiele levels, -Additional Points, Triangle Sorting Pieces, Pages 1-4 Sample Student Sorts 1-6	Paper triangles
Quadrilaterals and Their Properties	Quadrilateral Sort	20	6.14, 7.9	Quadrilateral Sorting Pieces	Paper quadrilaterals
	What's My Rule?	23	6.14, 7.9	What's My Rule?	Paper quadrilaterals
	Quadrilateral Properties Laboratory	25	6.14, 7.9	Types of Quadrilaterals	Geo-strips, D-stix, or miniature marshmallows and toothpicks; square corner
	Quadrilateral Sorting Laboratory	29	6.14, 7.9	Quadrilateral Sorting Laboratory, Quadrilateral Table, Quadrilateral Family Tree	Paper quadrilaterals
Plane Figures and Their Properties	Polygons and the Geoboard	35	6.14, 6.15, 7.9, 7.10		Geoboards, rubber bands



GEOMETRY

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
	A Country Mile	36	6.11, 6.14, 7.7, 7.9, 7.10	A Country Mile Writing Prompt	Geoboards, rubber bands
	Similarity and Congruence with Geostrips	38	6.14, 6.15, 7.9, 7.11		Geostrips, protractors, or angle rulers
	Perimeters and Areas of Similar Triangles	42	6.11, 7.7, 7.11	Data Chart: Area and Perimeter of Similar Triangles	Dynamic geometry software program
	Making and Using a Hypsometer	47	6.15, 7.11		Straws, decimal graph paper, cardboard, thread, a small weight, tape, hole punch, scissors, meter stick
	Human Circle	49	6.14		Pieces of string (5-10 feet) for each participant, chalk



Topic: The van Hiele Theory of Geometric Thought

Description: The van Hiele theory of geometric thought describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. In this first session, the participants will explore the van Hiele levels of geometric thought by doing triangle sorts and comparing their sorts to those performed by elementary students. The sorting task is appropriate for all ages and levels of students. It can serve as an activity to help students advance their level of understanding as well as an assessment tool that can inform the teacher at what van Hiele level the student is thinking.

Related SOL: 6.14, 6.15, 7.9, 7.10, 7.11, 7.12



Activity: The van Hiele Levels of Geometric Thought

Format: Large Group Lecture and Small Group Activity

Objectives: Participants will be able to describe the developmental sequence of geometric thinking according to the van Hiele theory of geometric thought and activities suitable for each level. In addition, participants will be able to assess the van Hiele levels of their students.

Related SOL: 6.14, 6.15

Materials: Paper triangles, cut out and placed in a plastic baggy or manila envelope (see Activity Sheet for Triangle Sorting Pieces). You will need at least one set of triangles for every three participants.

Time Required: Approximately 1 hour

Background: To Trainer (for lecture):
After observing their own students, Dutch teachers P.M. van Hiele and Dina van Hiele-Geldof described learning as a discontinuous process with jumps that suggest "levels." They identified five sequential levels of geometric understanding or thought:

- 1) Visualization
- 2) Analysis
- 3) Abstraction
- 4) Deduction
- 5) Rigor

Clements and Battista (1992) proposed the existence of a Level 0 that they called Pre-recognition.

In Kindergarten through grade two most students will be at Level 1. By grade three, students should be transitioning to Level 2. If the content in the Virginia Standards of Learning is mastered, students should attain Level 3 by the end of sixth grade. Level 4 is usually attained by students who can prove theorems using deductive techniques. One problem is that most current textbooks provide activities requiring only Level 1 thinking up through sixth grade and teachers must provide different types of tasks to facilitate the development of the higher levels of thought.

Directions: 1) Participants should study the The van Hiele Levels Explanation Sheet.



- 2) Turn to the Additional Points Explanation Sheet. Note for Point 1 that the levels are hierarchical. Students cannot be expected to write a geometric proof successfully unless they have progressed through each level of thought in turn. At Point 2, college students and even some teachers have been found who are at Level 1, while there are middle schoolers at Level 3 and above. (If the content in the SOL is mastered, students should attain Level 3 by the end of sixth grade.) As an example of an experience that can impede progress (Point 3), think of the illustration of the teacher who knew that the relationship between squares and rectangles was a difficult one for her fourth-graders so she had them memorize, "Every square is a rectangle, but not every rectangle is a square." When tested a few weeks later, half the students remembered that a square is a type of rectangle, while the other half thought that a rectangle was a type of square. It was almost impossible for these students to learn the true relationship between squares and rectangles because every time they heard the words square and rectangle together, they insisted on relying on their memorized sentence rather than on the properties of the two types of figures.
- 3) Continue on to Properties of Levels. As an example of separation, consider the meaning of the word "square." When a teacher thinking at Level 3 or above says "square," the word conveys the properties and relationships of a square: having four congruent sides; having four congruent angles; having perpendicular diagonals; and being a type of polygon, quadrilateral, parallelogram, and rectangle. To a student thinking at Level 1, the word "square" will only evoke an image of something that looks like a square such as a CD case or first base. The same word is being used, but it has an entirely different meaning to the teacher and the student. The teacher must keep in mind what the meaning of the word or symbol is to the student and how the student thinks about it. For Attainment, it is important to note that there are five phases of learning that lead to understanding at the next higher level.
- 4) Divide participants into small groups. Distribute the sets of cutout triangles, at least one set per three participants. Instruct the participants to lay out the pieces with the letters up. Do not call them triangles. Tell the participants that the objects can be grouped together in many different ways. For example, if we sorted the figures that make up the American flag (the red stripes, the white stripes, the blue field, the white stars), we might sort by color and put the white stripes and the stars together because they



are white, the red stripes in another group because they are red, and the blue field by itself because it is the only blue object. Another way to sort the flag parts would be to put all the stripes and the blue field together because they are all rectangles and all the stars together because they are not rectangles. If needed, you can demonstrate a triangle sort using pieces cut from the Triangle Sorting Pieces Activity Sheet. Have participants sort the figures into groups that belong together, recording the letters of the pieces they put together and the criteria they used to sort. Have them sort two or three times, recording each sort.

- 5) Ask the participants to describe their sorts. Expect answers like "acute, right, and obtuse triangles" or "scalene, isosceles, and equilateral." Have them compare their sorts with those of other groups.
- 6) Ask them how they think their students would sort these figures. Refer to Sample Student Sort Sheets and ask the participants to conjecture the criteria used for sorting and the van Hiele level of the sorter. Sample Student Sort 1 is a low Level 1 sort where the student is sorting strictly by size and may not even know that the figures are triangles. Sample Student Sort 2 is another Level 1 sort. Here the student thinks that triangles must have at least two sides the same length or possibly that triangles must be symmetric. Sample Student Sort 3 is another Level 1 sort. This student also believes that triangles must have at least two sides the same length or possibly that triangles must be symmetric. Additionally, this student recognized the figures with right angles or "corners" as a separate category. The Sample Student Sort 4 is at least a Level 2 or 3 sort in which the sorter focuses on the lengths of the sides, a criterion that separates the figures into categories that overlap. The student has actually sorted into groups with no sides the same length, two sides the same length, and all sides the same length. It is unclear whether the student knows that equilateral triangles are a type of isosceles triangle. The Sample Student Sort 5 focuses on parts of the figures and so is a Level 2 sort, but the student does not have the vocabulary to adequately describe the figures. The Sample Student Sort 6 is similar to the Sample Student Sort 4, but the word "Perfect" is incorrect and indicates that the student may be thinking more of the figure as a whole rather than of the individual parts. This sort is probably Level 2

Note: Descriptions written on the Sample Student Sort Sheets are the words of the student who performed the sort.



Explanation Sheet: The van Hiele Levels

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of the figure. A student should recognize and name figures, and distinguish a given figure from others that look somewhat the same. "I know it's a rectangle because it looks like a door and I know that the door is a rectangle."

Level 2: Analysis. Properties are perceived, but are isolated and unrelated. A student should recognize and name properties of geometric figures. "I know it's a rectangle because it is closed, it has four sides and four right angles, opposite sides are parallel, opposite sides are congruent, diagonals bisect each other, adjacent sides are perpendicular,..."

Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. "I know it's a rectangle because it's a parallelogram with right angles."

Level 4: Deduction. The student can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. A student should be able to supply reasons for steps in a proof.

Level 5: Rigor. The standards of rigor and abstraction represented by modern geometries characterize Level 5. Symbols without referents can be manipulated according to the laws of formal logic. A student should understand the role and necessity of indirect proof and proof by contrapositive.



Explanation Sheet:

Additional Points

1. The learner cannot achieve one level without passing through the previous levels.
2. Progress from one level to another is more dependent on educational experience than on age or maturation.
3. Certain types of experiences can facilitate or impede progress within a level or to a higher level.

Properties of Levels

Adjacency: What was intrinsic in the preceding level is extrinsic in the current level.

Distinction: Each level has its own linguistic symbols and its own network of relationships connecting those symbols.

Separation: Two individuals reasoning at different levels cannot understand one another.

Attainment: The learning process leading to complete understanding at the next higher level has five phases: inquiry/information, directed orientation, explication, free orientation, and integration.

Phases of Learning

Inquiry/Information: Gets acquainted with the working domain (e.g., examines examples and non-examples).

Guided orientation: Does tasks involving different relations of the network that is to be formed (e.g., folding, measuring, looking for symmetry).

Explication: Becomes conscious of the relations, tries to express them in words, and learns technical language which accompanies the subject matter (e.g., expresses ideas about properties of figures).

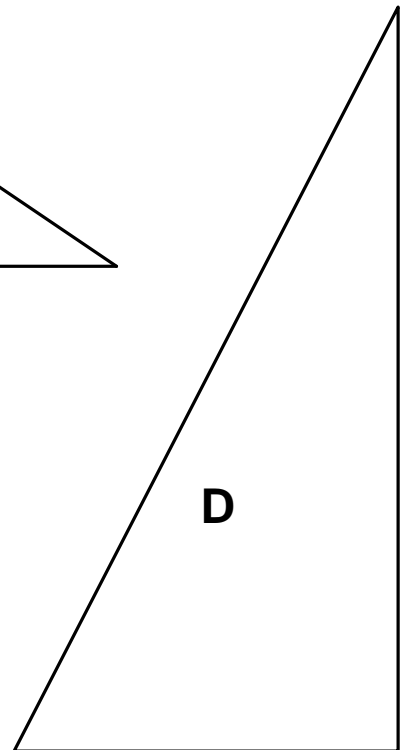
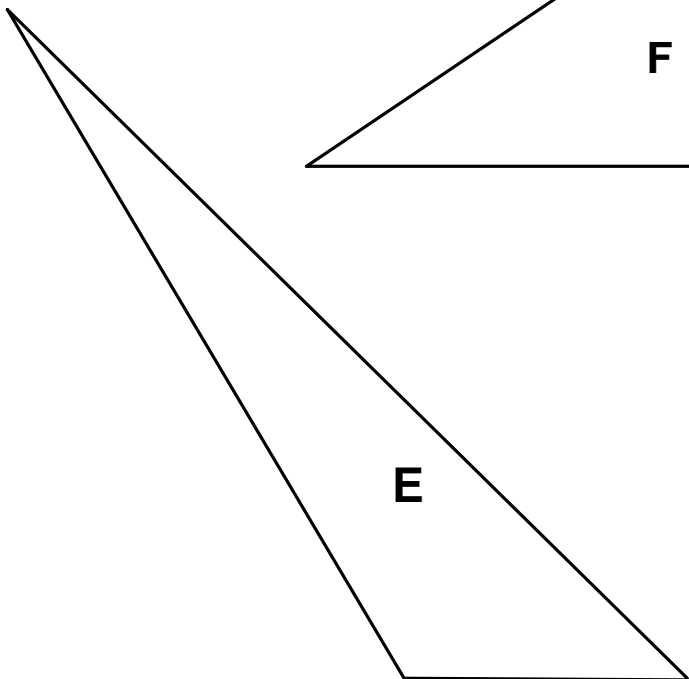
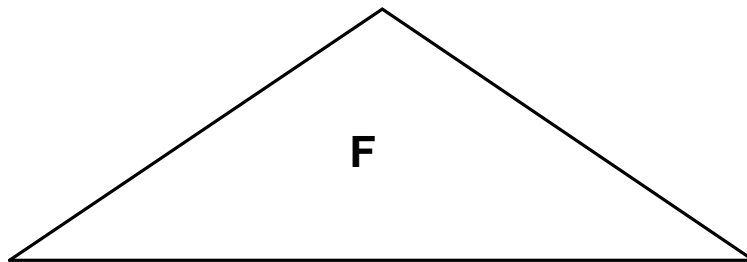
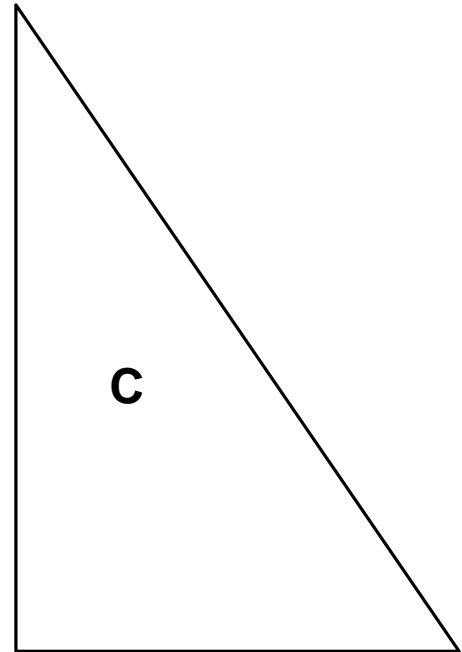
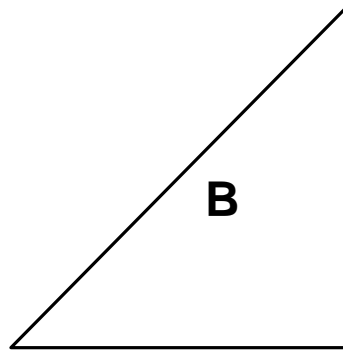
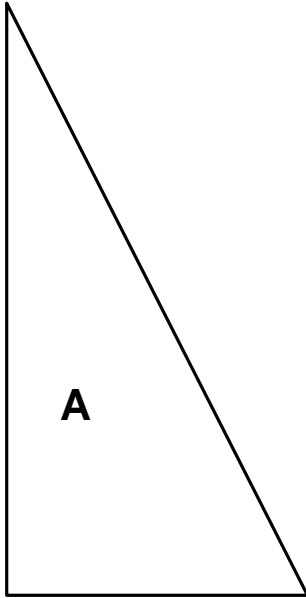
Free orientation: Learns, by doing more complex tasks, to find his/her own way in the network of relations (e.g., knowing properties of one kind of figure, investigating these properties for a new figure, such as kites).

Integration: Summarizes all that has been learned about the subject, then reflects on actions, and obtains an overview of the newly formed network of relations now available (e.g., properties of a figure are summarized).



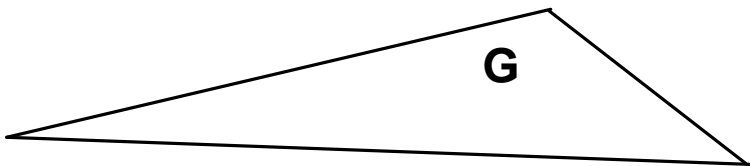
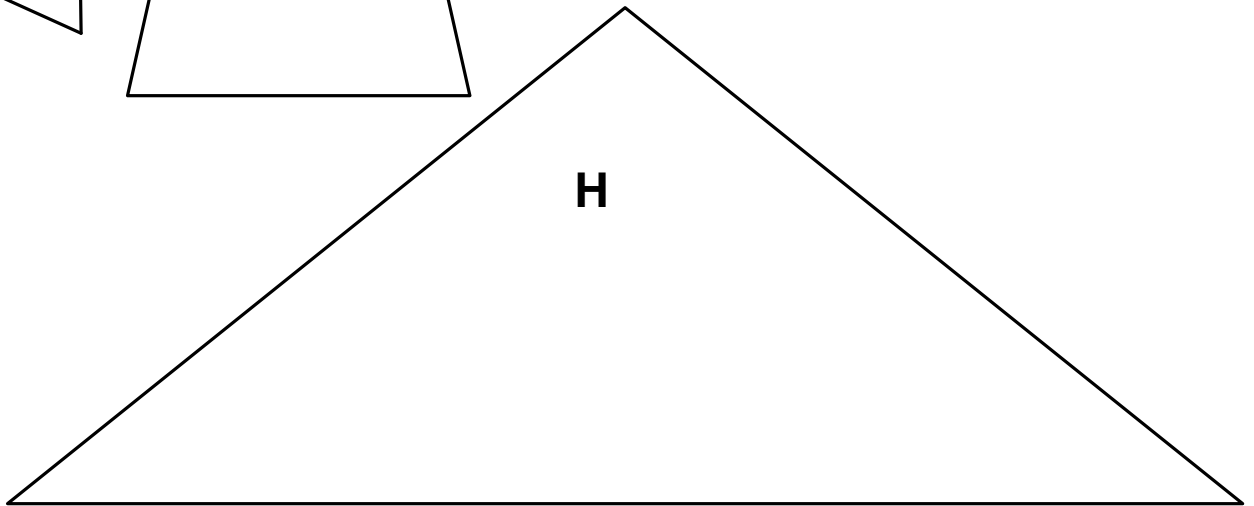
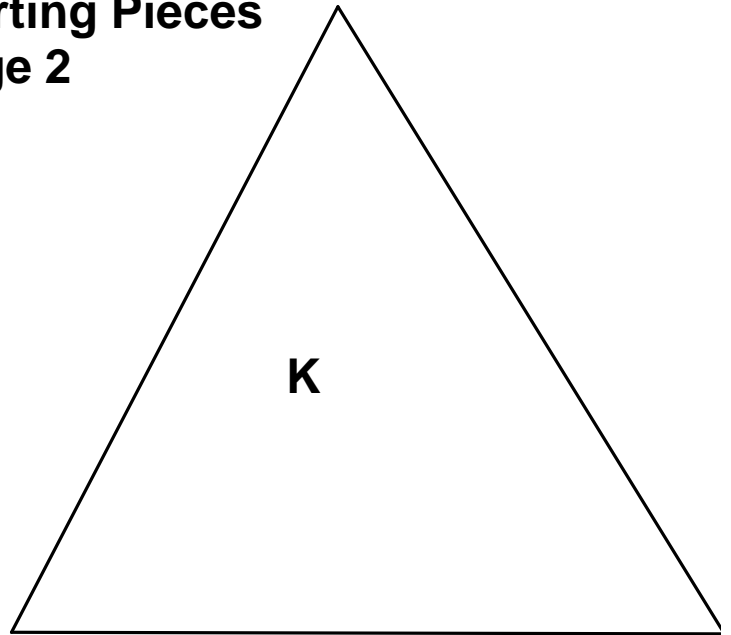
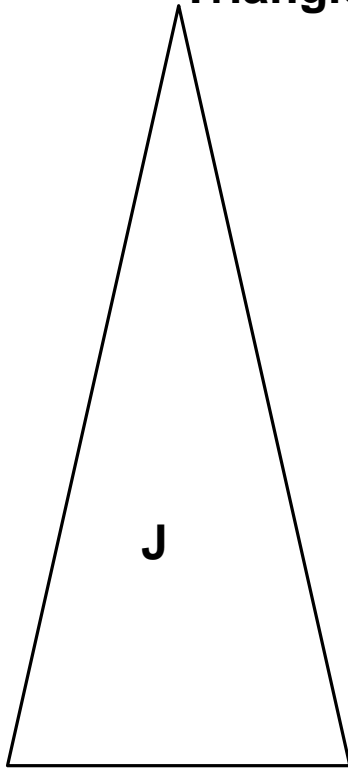
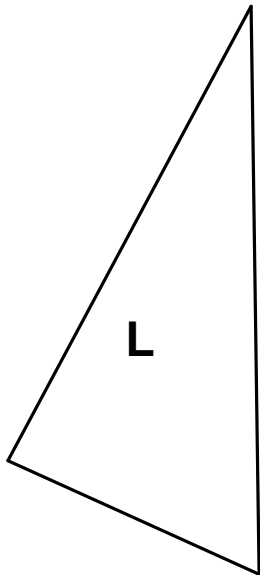
Triangle Sorting Pieces

Page 1



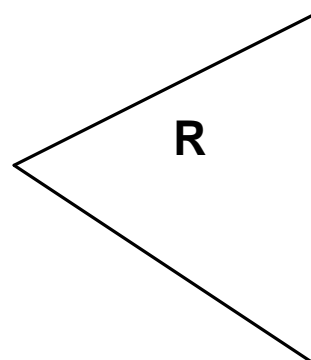
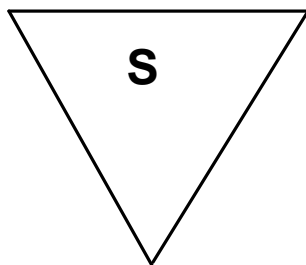
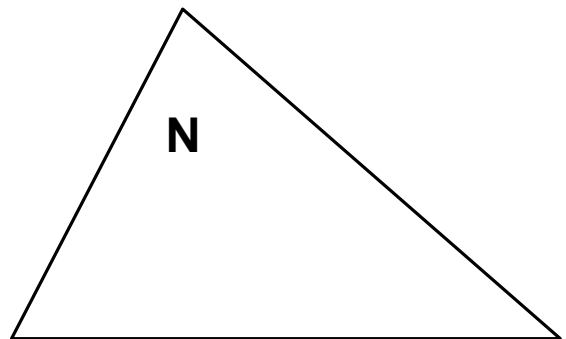
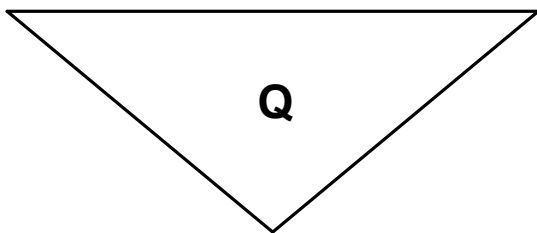
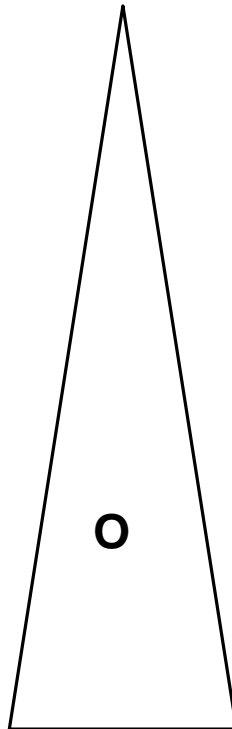
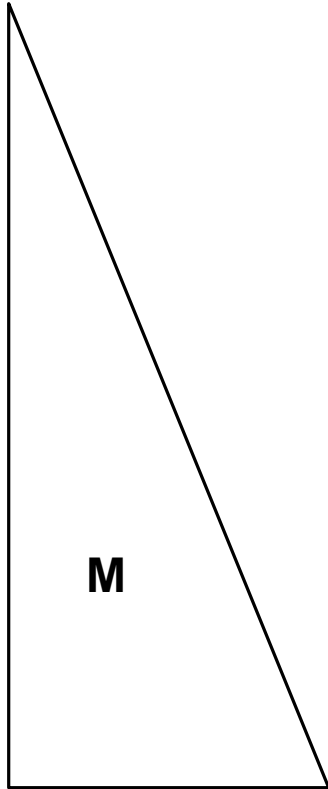


Triangle Sorting Pieces Page 2



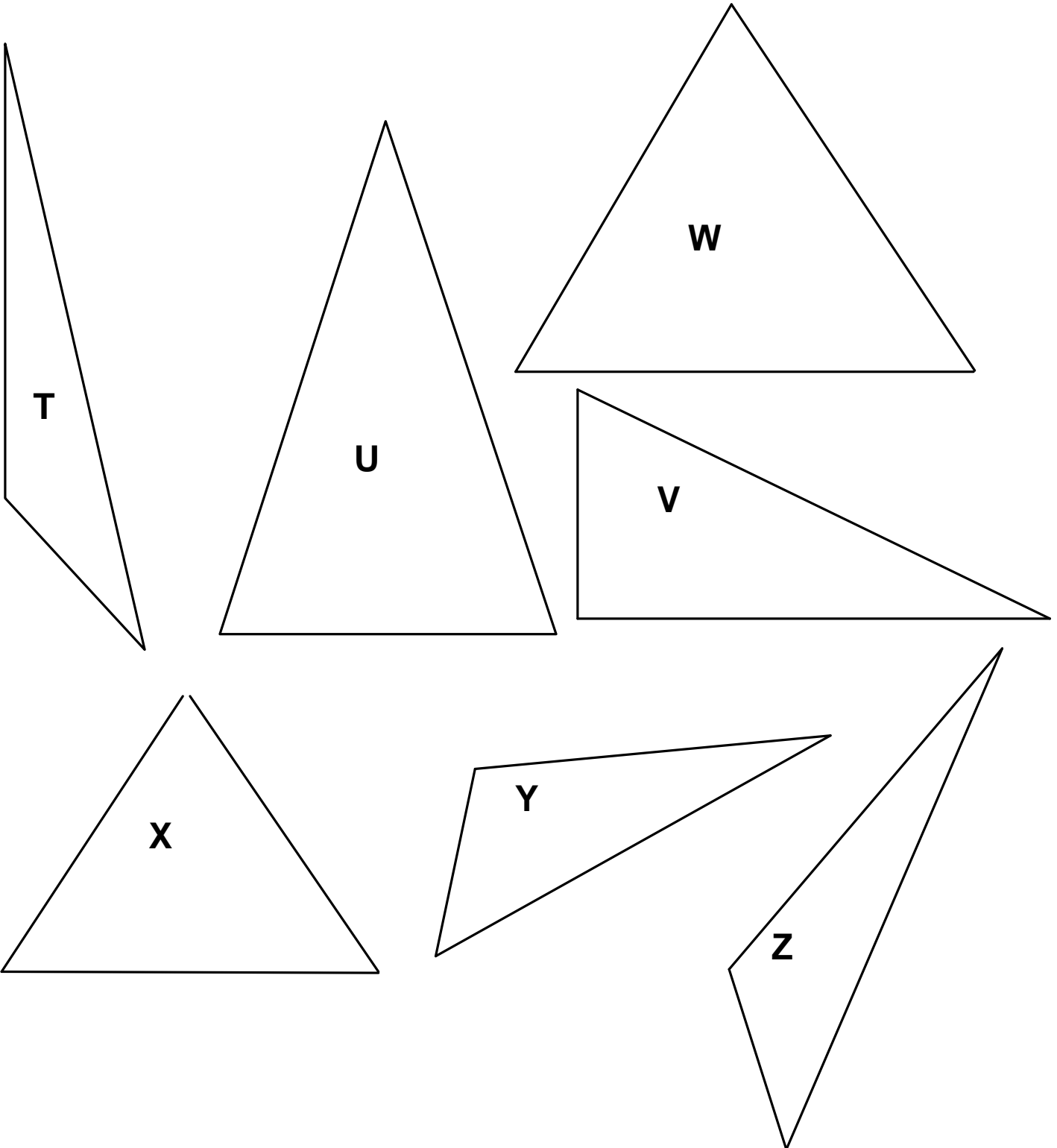


Triangle Sorting Pieces Page 3



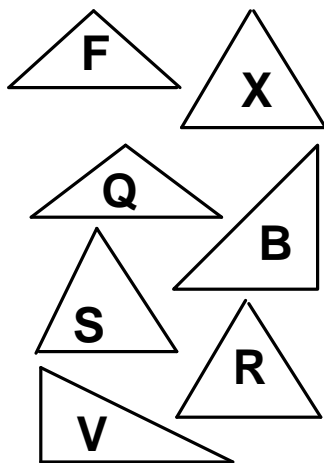


Triangle Sorting Pieces Page 4

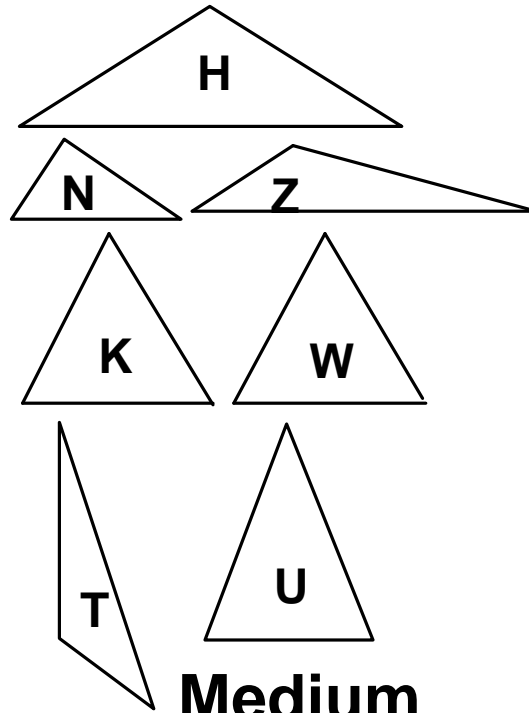




Sample Student Sort 1

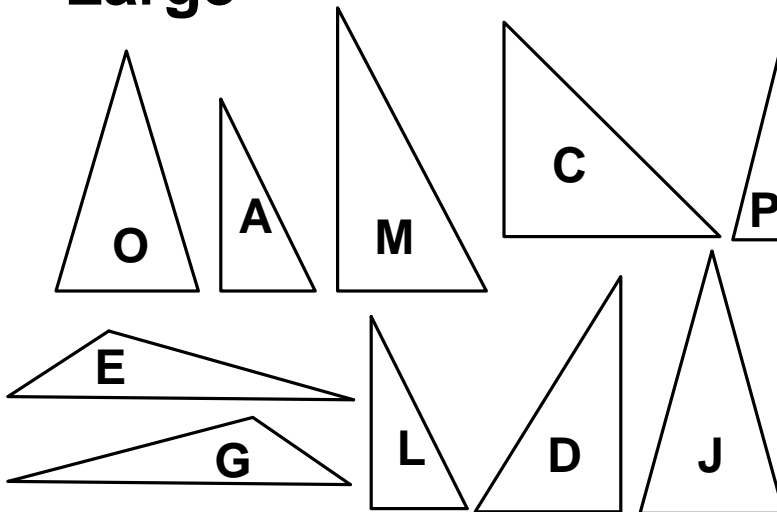


Small



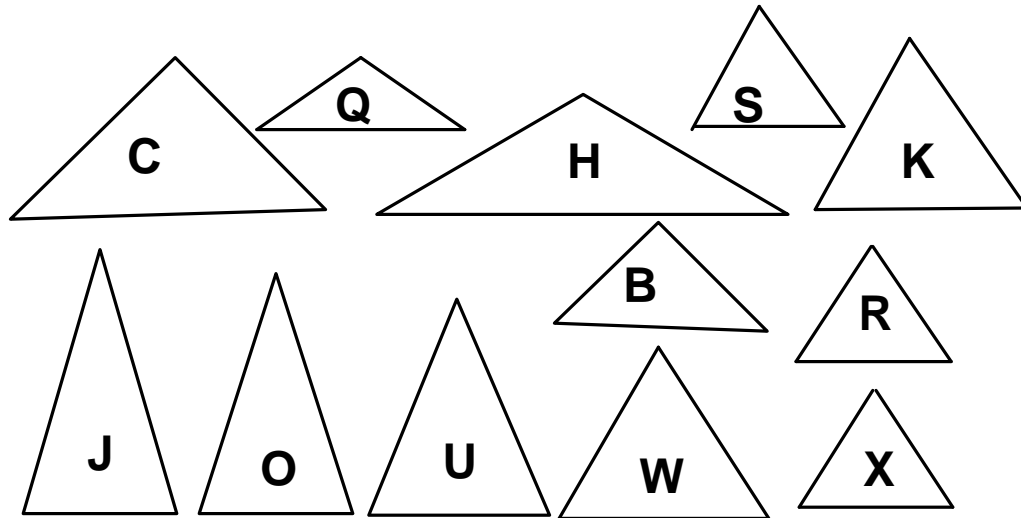
Medium

Large

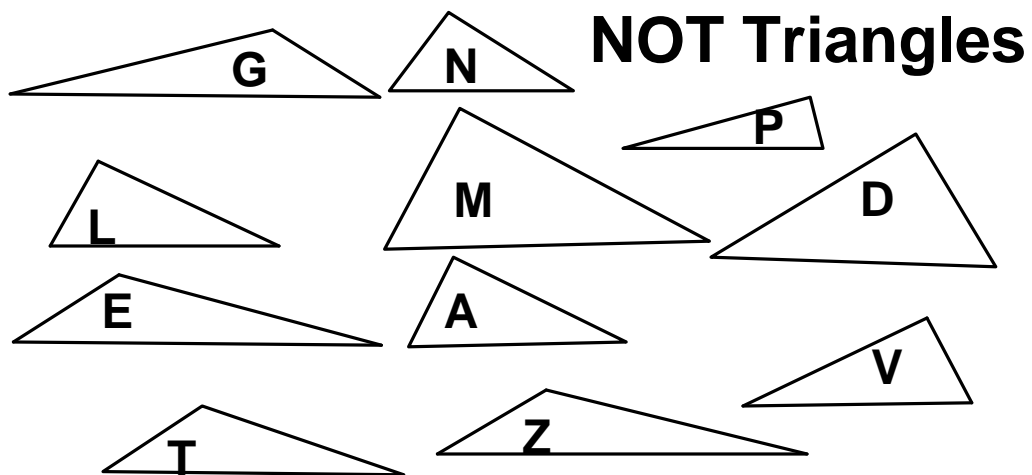




Sample Student Sort 2



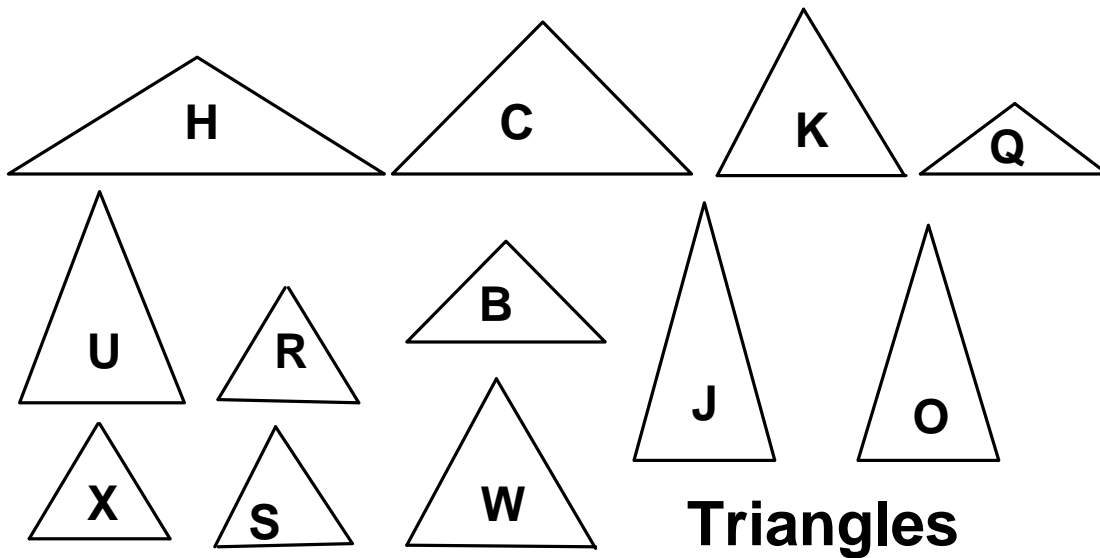
Triangles



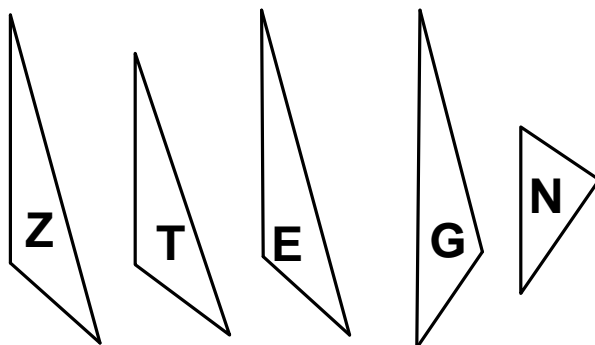
NOT Triangles



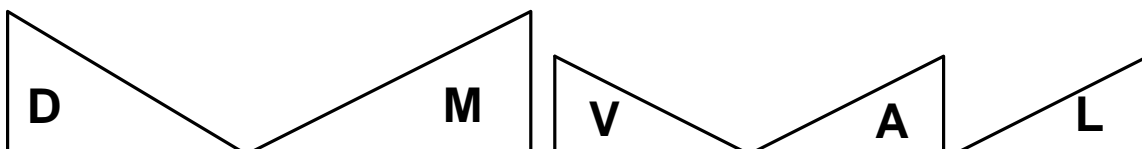
Sample Student Sort 3



Triangles



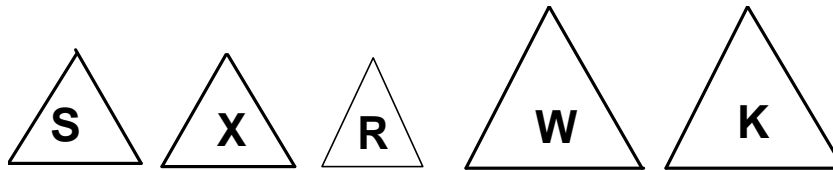
**Look alike...
N is smaller...
NOT triangles**



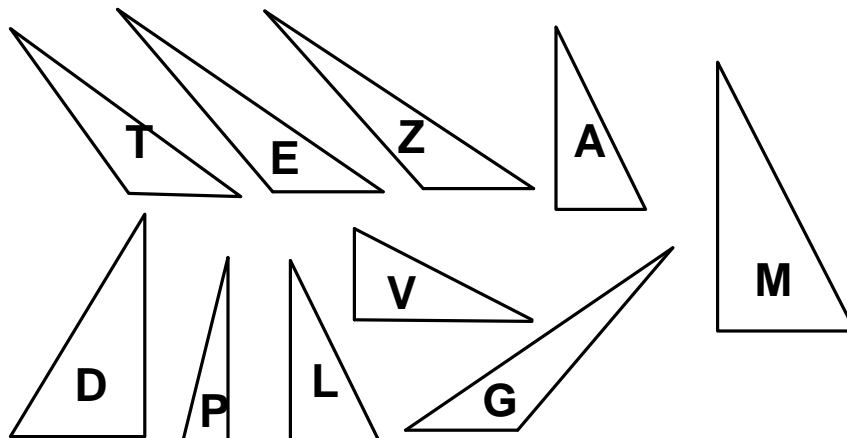
Look like ramps... NOT triangles



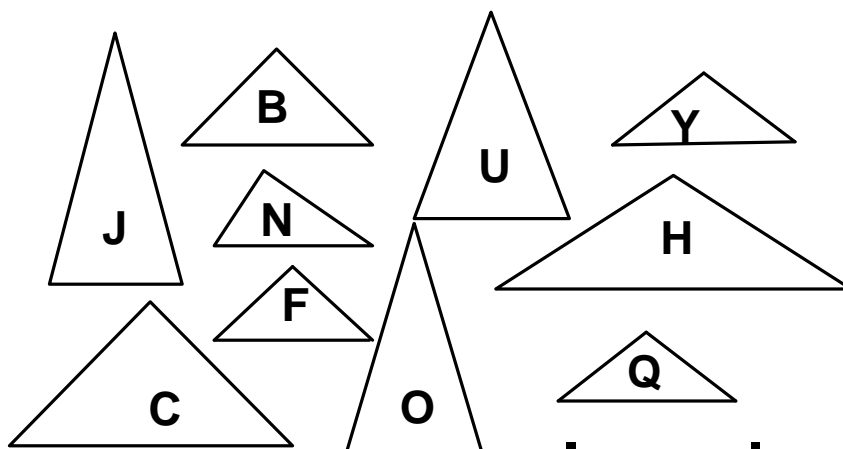
Sample Student Sort 4



Equilateral



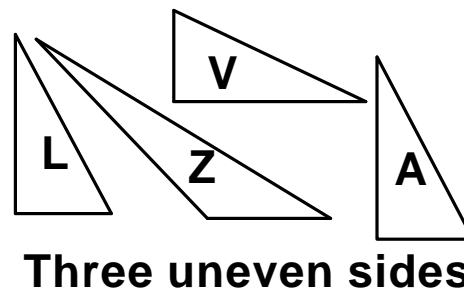
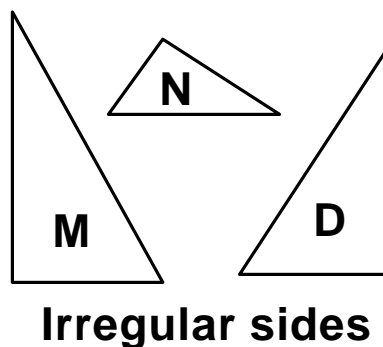
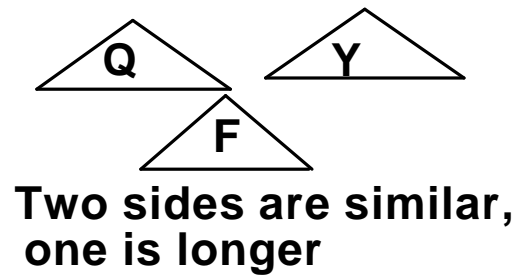
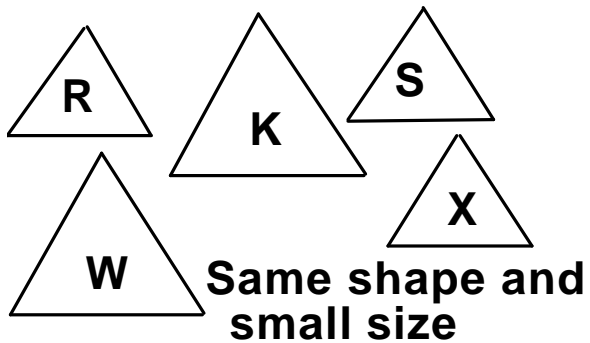
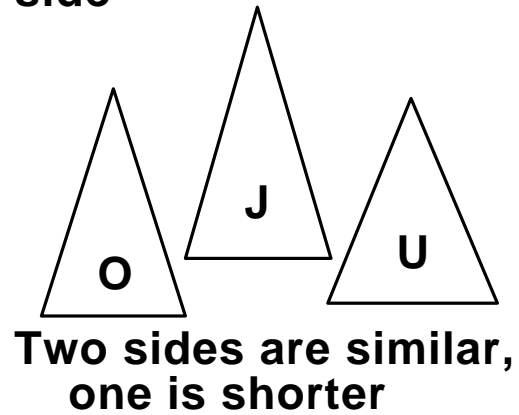
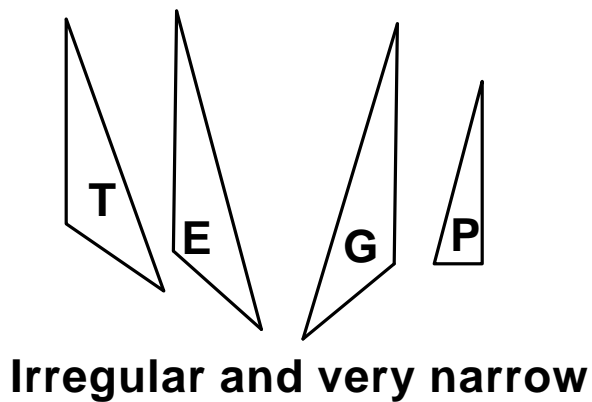
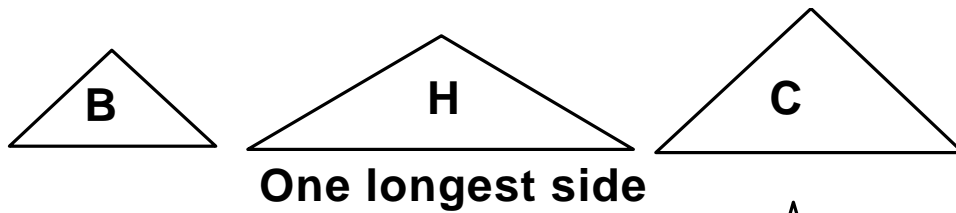
Scalene



Isosceles

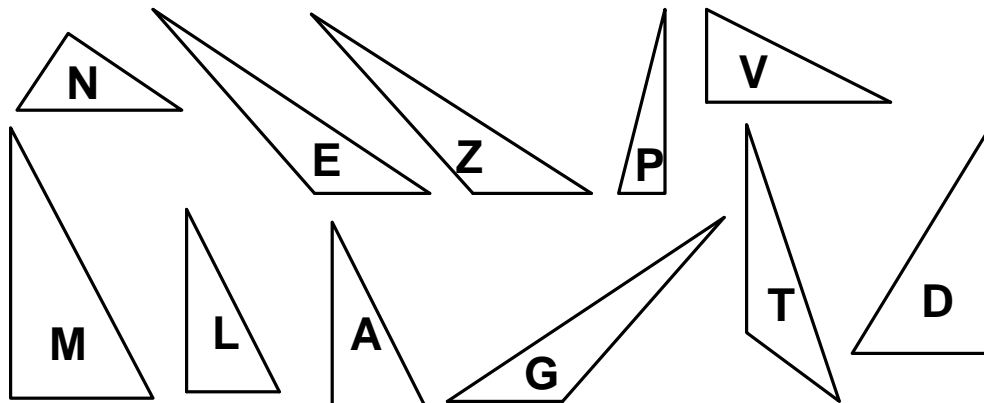


Sample Student Sort 5

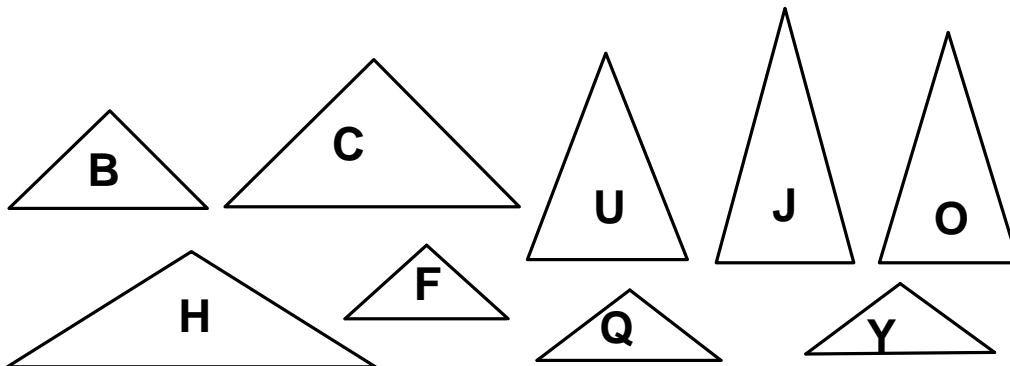




Sample Student Sort 6

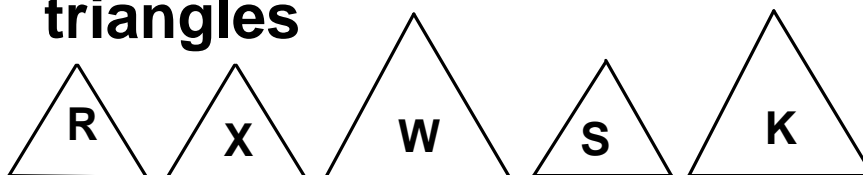


Every side has a different size



Two sides equal, third side smaller or larger

**“Perfect”
triangles**





Topic: Quadrilaterals and their Properties

Description: Participants will explore quadrilaterals and their properties through the use of various manipulatives such as sorting pieces and geo-strips. The sequence of activities is designed to facilitate an increase in a learner's van Hiele level of thinking about quadrilaterals from Level 1 to Level 3. First, the participants learn how to determine the van Hiele levels of their own students by analyzing how they sort a set of quadrilateral pieces. Then they play the game "What's My Rule?" to develop the ability to classify quadrilaterals by various attributes and to focus on more than one attribute at a time. The participants also construct parallelograms, rectangles, rhombi, and squares using D-stix, geo-strips, toothpicks, or other manipulatives and make observations while the figures are flexed (Level 2). Finally, the participants identify relationships between parallelograms, rectangles, rhombi, squares, trapezoids, kites, and darts through a lab that culminates in the creation of a quadrilateral family tree (Level 3).

Although these activities are presented with quadrilaterals, most of them are easily adapted to triangles and other polygons.

Related SOL: 6.14, 7.9



Activity: Quadrilateral Sort

Format: Small Group/Large Group

Objectives: After performing their own sorts, participants will be able to distinguish the way students at various van Hiele levels of geometric thought may sort quadrilaterals.

Related SOL: 6.14, 7.9

Materials: Quadrilateral Sorting Pieces Activity Sheet with quadrilaterals cut out and placed in a plastic baggy or manila envelope. You will need at least one set of quadrilaterals for every three participants.

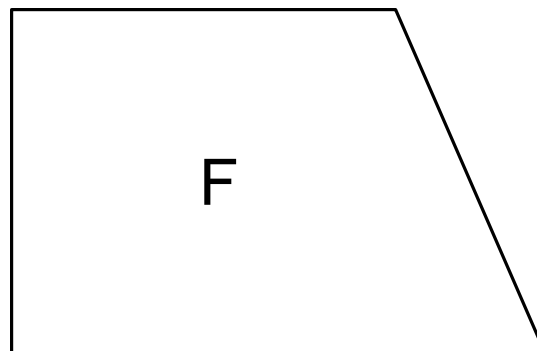
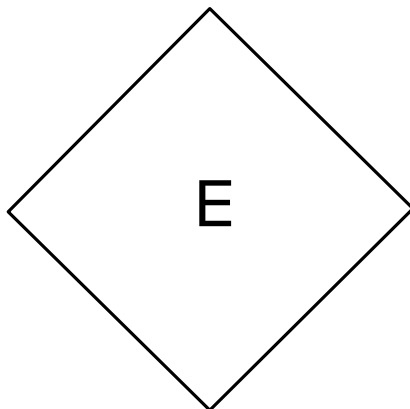
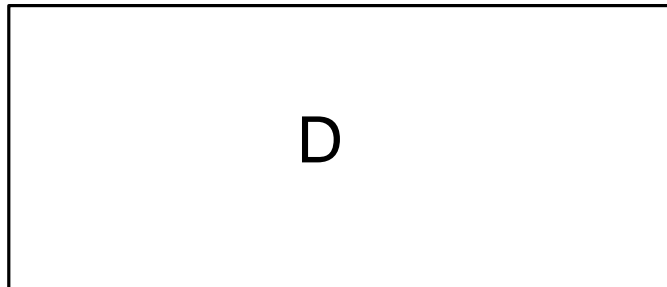
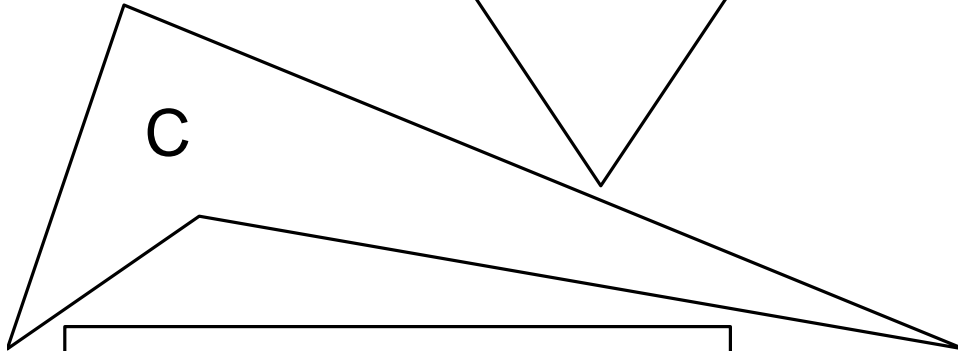
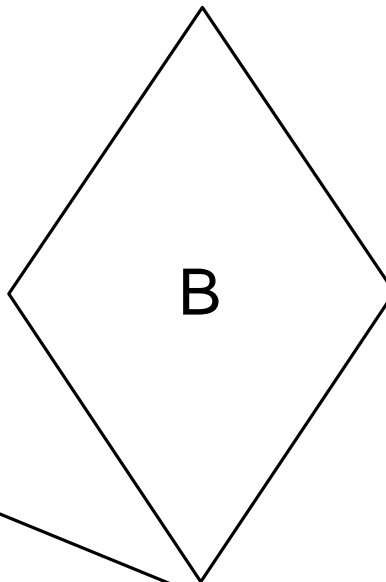
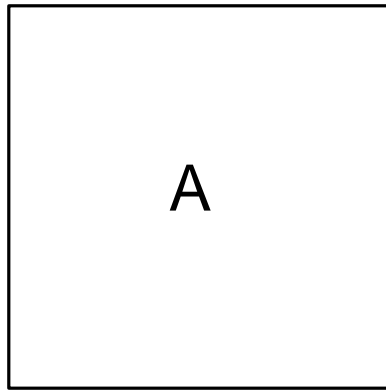
Time Required: Approximately 20 minutes

Directions:

- 1) Divide the participants into small groups. Distribute the sets of cut-out quadrilaterals, at least one set per three participants. Instruct the participants to lay out the pieces with the letters up. Do not call them quadrilaterals. Tell the participants that the objects can be grouped together in many different ways. For example, if we sorted the figures that make up the American flag (the red stripes, the white stripes, the blue field, the white stars), we might sort by color and put the white stripes and the stars together because they are white, the red stripes in another group because they are red, and the blue field by itself because it is the only blue object. Another way the flag parts could be grouped would be all the stripes and the blue field together because they are all rectangles and all the stars together because they are not rectangles. Have them sort the figures into groups that belong together, recording the letters of the pieces they put together and the criteria they used to sort. Have them sort two or three times, recording each sort.
- 2) Ask the participants to describe their sorts. Have them compare their sorts with those of other groups.
- 3) Ask them how they think their students would sort these figures.

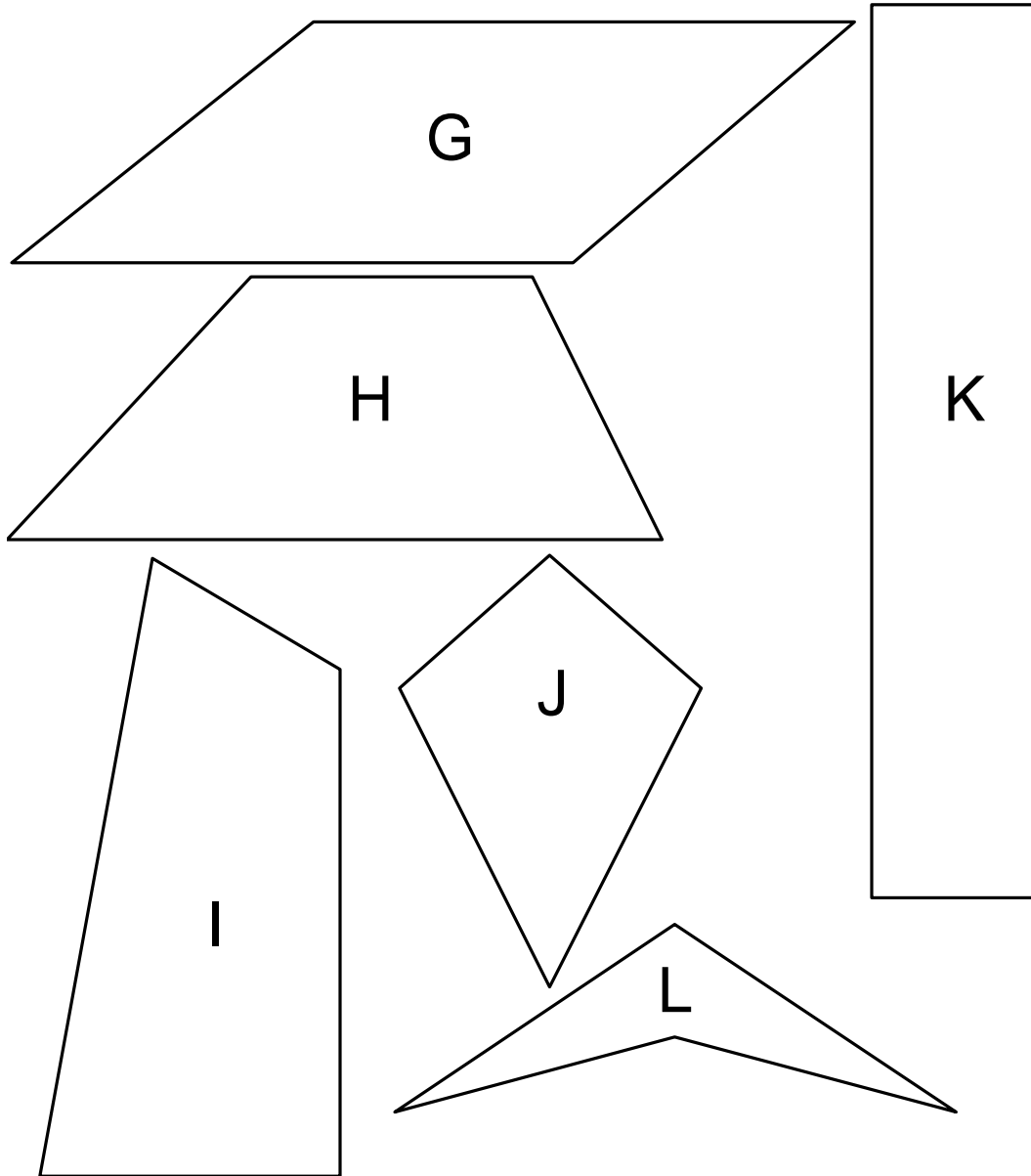


Quadrilateral Sorting Pieces





Quadrilateral Sorting Pieces Page 2





Activity: What's My Rule?

Format: Small Group/Large Group

Objectives: After playing the game, participants will classify quadrilaterals by various attributes. In children, this game develops the ability to attend to more than one characteristic of a figure at the same time.

Related SOL: 6.14, 7.9

Materials: Quadrilateral Sorting Pieces Activity Sheet with quadrilaterals cut out and placed in a plastic baggy or manila envelope. (Use quadrilaterals from Quadrilateral Sort Activity). You will need at least one set of quadrilaterals for every three or four participants. "What's My Rule?" Activity Sheet

Time Required: Approximately 10 minutes

Directions:

- 1) Divide the participants into small groups. Distribute the sets of cut-out quadrilaterals, one set per group.
- 2) Display "What's My Rule?" Activity Sheet and review the rules of the game. One participant in each group is the sorter. The sorter writes down a "secret rule" to classify the set of quadrilaterals into two or more piles and uses that rule to slowly sort the pieces as the other players observe.
- 3) At any time, the players can call "stop" and guess the rule. After the correct rule identification, the player who figured out the rule becomes the sorter. The correct identification is worth five points. A correct answer, but not the written one, is worth one point. Each incorrect guess results in a two-point penalty. The winner is the first one to accumulate ten points.



WHAT'S MY RULE?

Rules

1. Choose one player to be the sorter. The sorter writes down a "secret rule" to classify the set of quadrilaterals into two or more piles and uses that rule to slowly sort the pieces as the other players observe.
2. At any time, the players can call "stop" and guess the rule. The correct identification is worth five points. A correct answer, but not the written one, is worth one point. Each incorrect guess results in a two-point penalty.
3. After the correct rule identification, the player who figured out the rule becomes the sorter.
4. The winner is the first one to accumulate ten points.



Activity: **Quadrilateral Properties Laboratory**

Format: Small Group/Large Group

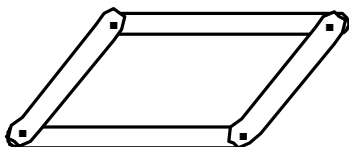
Objectives: Participants will construct parallelograms, rectangles, rhombi, and squares, using D-stix, geo-strips, or toothpicks and marshmallows. Participants will identify the properties of the constructed figures.

Related SOL: 6.14, 7.9

Materials: One of the following per participant: D-stix, geo-strips, or toothpicks cut into two different lengths and marshmallows; square corner (the corner of an index card or book); Types of Quadrilaterals Activity Sheet

Time Required: Approximately 20 minutes

- Directions:**
- 1) Divide the participants into small groups and direct each group to experiment as you ask questions. Be sure to model constructing the quadrilaterals and flexing them.
 - 2) Have the participants pick two pairs of congruent segments and connect them as shown below. Have them flex the figure to different positions.



Ask:

- What stays the same? (Lengths of the sides, the opposite sides are parallel, opposite angles are congruent, sum of the measures of the angles, perimeter)
- What changes? (Size of angles, area, lengths of diagonals)
- What do you notice about the opposite sides of this quadrilateral? (They remain parallel and congruent.)

A **parallelogram** is a quadrilateral with both pairs of sides parallel.

- What is the sum of the measures of the interior angles of this quadrilateral? (360°)



- What do you notice about the opposite angles? (Congruent)

Note to Trainer: Some participant will likely turn the strips so that they cross, forming two triangles. If no one does, you should. Ask if this figure is a polygon. Elicit from the group what the essential elements of a polygon are, i.e.:

- a) composed of line segments;
- b) simple (the segments don't cross);
- c) closed; and
- d) lies in a plane (e.g., if you take a wire square and twist it so that it isn't flat, it is no longer a polygon).

- 3) Make one of the angles a right angle. You can use the square corner to check your accuracy.

Ask:

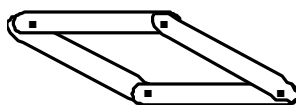
- What happens to the other angles? (They become right angles.)
- Will this always be true when you make one angle of a parallelogram a right angle? (Yes)
- How do you know? (The sum of the measure of the angles in a parallelogram is 360° . One angle measures 90° . Its opposite angle must measure the same or 90° . Subtracting these two angles from 360° , the remaining two angles, which are congruent since they are opposite angles in a parallelogram, must have a total measure of 180° . Therefore, each angle measure is 90° . Note: This is Level 3 thinking.)
- Is it still a parallelogram? (Yes)
- Is it still a quadrilateral? (Yes)
- Is it still a polygon? (Yes)
- What other name, besides polygon, quadrilateral, and parallelogram, can be given to it now? (Rectangle)

A **rectangle** is a parallelogram with four right angles.

- 4) Make a parallelogram that has all four sides equal in length. What is another name for this parallelogram? (Rhombus)

A **rhombus** is a parallelogram with four congruent sides.

- 5) Flex the figure to different positions.



Ask:

- What stays the same? (Lengths of the sides, the opposite sides are parallel, opposite angles are congruent, sum of angles, perimeter)
- What changes? (Size of angles, area, lengths of diagonals)
- What is the sum of the measures of the interior angles of this quadrilateral? (360°)
- What do you notice about the opposite angles? (Congruent)
- Is it still a quadrilateral? (Yes)
- Is it still a polygon? (Yes)

- 6) Make one of the angles of this rhombus a right angle, checking with your square corner.

Ask:

- What happens to the other angles? (All right angles)
- Is it still a parallelogram? (Yes)
- What other name, besides polygon, quadrilateral, parallelogram, and rhombus, can be given to this new figure? (Square)

A **square** is a parallelogram with four congruent sides and four right angles.

Ask:

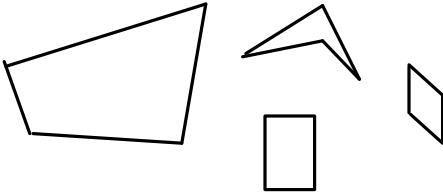
- Is it a rectangle? (Yes)
- How do you know? (It has four right angles.)

- 7) Distribute Types of Quadrilaterals Activity Sheet and discuss the definitions for quadrilateral, parallelogram, rectangle, rhombus, and square. Discuss the examples of each, noticing their orientations and how each example fits the definition even though they are not necessarily the stereotypical figure usually seen. Discuss the implications for teaching a Level 1 student who recognizes figures by comparing them to a known figure. This type of student might describe a rectangle by saying, "I know it's a rectangle because it looks like a door."

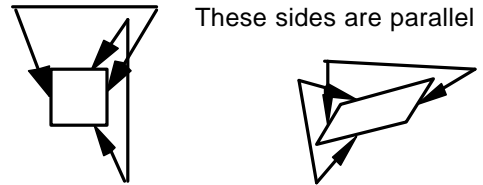


TYPES OF QUADRILATERALS

A **quadrilateral** is a **four-sided polygon**.

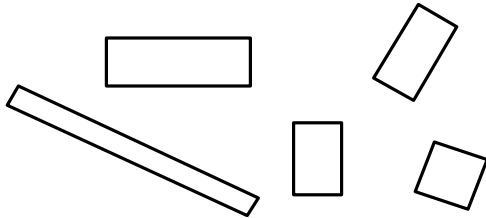


A **parallelogram** is a quadrilateral with both pairs of **opposite sides parallel**.

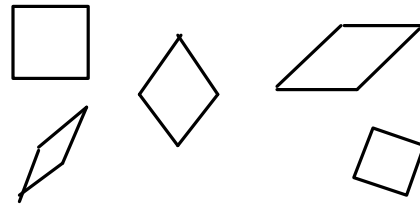


These sides are parallel

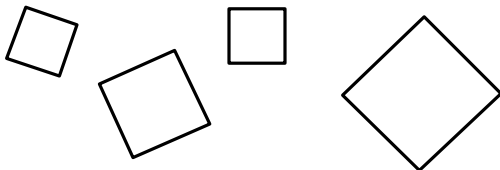
A **rectangle** is a quadrilateral with **four right angles**.



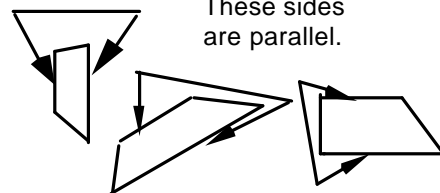
A **rhombus** is a quadrilateral with **four sides congruent**.



A **square** is a quadrilateral with **four right angles** and **four congruent sides**.

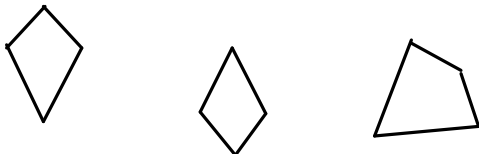


A **trapezoid** is a quadrilateral with **exactly one pair of parallel sides**.

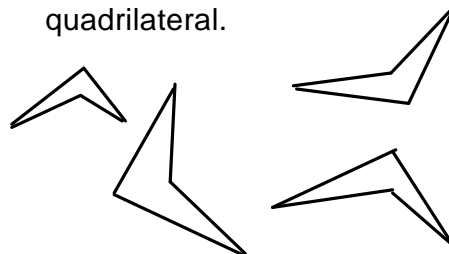


These sides are parallel.

A **kite** is a convex quadrilateral with two distinct pairs of **adjacent congruent sides**.



A **dart** is a **concave** quadrilateral.





Activity: Quadrilateral Sorting Laboratory

Format: Small Group/Large Group

Objectives: Participants will record which quadrilaterals meet the various descriptions listed in the properties table, determine which sets are identical and are subsets of one another, attach labels to each category, and create a quadrilateral family tree.

Related SOL: 6.14, 7.9

Materials: Quadrilateral Sorting Pieces Activity Sheet with quadrilaterals cut out and placed in a plastic baggy or manila envelope. (Use quadrilaterals from Quadrilateral Sort Activity). Quadrilateral Sorting Laboratory Activity Sheet, Quadrilateral Table Activity Sheet, Quadrilateral Family Tree Activity Sheet

Time Required: Approximately 30 minutes

Directions:

- 1) Distribute Quadrilateral Sorting Laboratory Activity Sheet, Quadrilateral Table Activity Sheet and the Quadrilateral Family Tree Activity Sheet. Divide the participants into small groups and direct each group to experiment and answer the questions, using their quadrilateral sorting pieces.
- 2) After the participants have filled out the Quadrilateral Table Activity Sheet, have pairs of groups compare their answers, and reconcile any discrepancies.
- 3) Have the participants continue with Steps 5-10. Refer to the Quadrilateral Table as needed while discussing the results of #10.
- 4) For Step 11 the participants can construct the family tree as small groups or as a large group. Discuss various possibilities for the entries.



Quadrilateral Sorting Laboratory

- Directions:**
- 1) Spread out your quadrilateral pieces with the letters facing up so you can see them.
 - 2) Find all of the quadrilaterals having four right angles. List them by letter alphabetically in the corresponding row of the Quadrilateral Table.
 - 3) Consider all of the quadrilaterals again. Find all of the quadrilaterals having exactly one pair of parallel sides. List them by letter alphabetically in the corresponding row of the Quadrilateral Table.
 - 4) Continue in this manner until the Quadrilateral Table is complete.
 - 5) Which category is the largest? What name can be used to describe this category?
 - 6) Which lists are the same? What name can be used to describe quadrilaterals with these properties?
 - 7) Are there any lists that are proper subsets of another list? If so, which ones?
 - 8) Are there any lists that are not subsets of one another that have some but not all members in common? If so, which ones?
 - 9) Which lists have no members in common?
 - 10) Label each of the categories in the Quadrilateral Table with the most specific name possible using the labels kite, quadrilateral, parallelogram, rectangle, rhombus, square, and trapezoid. For example, #1 - a quadrilateral that has four right angles is a rectangle. (Having four right angles is not enough to make it a square; it would need four congruent sides as well.)
 - 11) Compare your results to that of the other groups. Then fill out the family tree by inserting the names kites, rectangles, squares, and trapezoids into the appropriate places on the diagram.

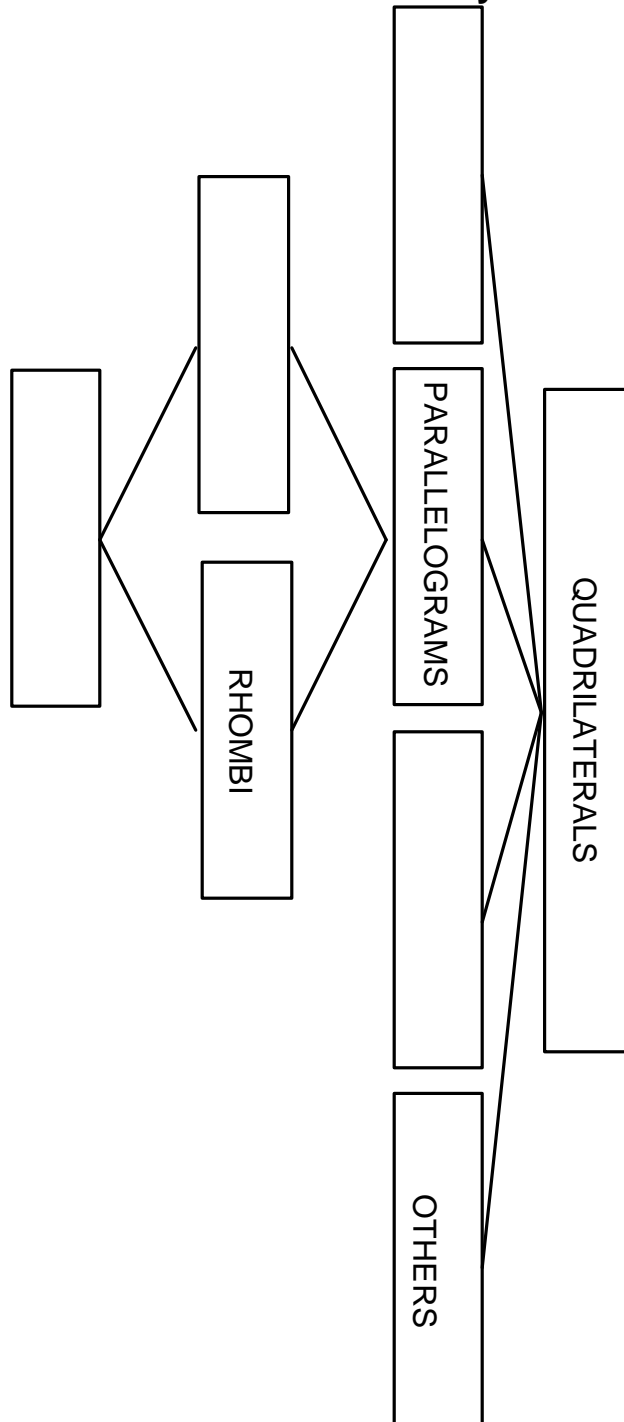


Quadrilateral Table

- | | |
|-----|--|
| 1. | has four right angles |
| 2. | has exactly one pair of parallel sides |
| 3. | has two pairs of opposite sides congruent |
| 4. | has four congruent sides |
| 5. | has two pairs of opposite sides parallel |
| 6. | has no sides congruent |
| 7. | has two pairs of adjacent sides congruent, but not all sides congruent |
| 8. | has perpendicular diagonals |
| 9. | has opposite angles congruent |
| 10. | is concave |
| 11. | is convex |
| 12. | its diagonals bisect one another |
| 13. | has four sides |
| 14. | has four congruent angles |
| 15. | has four congruent sides and four congruent angles |



Quadrilateral Family Tree





Topic: Plane Figures and their Properties

Description: Participants will explore plane figures and their properties through the use of various manipulatives such as geoboards, and geo-strips. A writing exercise is also included. Similar, congruent, and non-congruent plane figures are constructed and examined.

Related SOL: 6.11, 6.13, 6.14, 6.15, 7.7, 7.9, 7.10



Activity: Polygons and the Geoboard

Format: Large Group

Objectives: The participant will correctly manipulate the rubber band on the geoboard to make polygons with an increasing number of sides.

Related SOL: 6.14, 6.15, 7.9, 7.10

Materials: 11 x 11 geoboards, rubber bands

Time Required: Approximately 15 minutes

Directions:

- 1) Remind participants of appropriate use of the geoboards and ensure that everyone has a board and a rubber band. Ask participants to make a triangle on their geoboard.
- 2) Ask the participants to modify the shape on their geoboard by adding one more side. Continue from a triangle to a quadrilateral to a pentagon, and so forth.
- 3) Have the participants compare their shapes on the geoboards with each other. Use this opportunity to talk about the fact that all four-sided figures are quadrilaterals — some are squares, some trapezoids, and so forth. This is a good time to reinforce the idea of common properties (all quadrilaterals have four sides) and distinguishing properties (all squares have four congruent sides and four right angles.)
- 4) Challenge participants to make a polygon with the largest number of sides. Is the answer different if the polygon is convex or concave?



Activity: A Country Mile

Format: Individual or Small Group

Objectives: Participants will find and support a reasonable response to the writing prompt using their knowledge of the perimeter of polygons and circles.

Related SOL: 6.11, 6.14, 7.7, 7.9, 7.10

Materials: A Country Mile Writing Prompt Activity Sheet, geoboards and rubber bands (optional)

Time Required: Approximately 30 minutes plus writing time

Directions:

- 1) Discuss the nature of an open response item with participants. Let them know that these questions may have more than one correct answer. When used with students, work should be graded for coherence of thought, careful presentation, reasonableness of the answer, and support of the answer given.
- 2) The prompt as presented has very few hints included. The questions below may help you guide participants who have difficulty getting started.
 - How long is a day from sunup to sundown? Is it the same all year?
 - What shape includes the most area for a given perimeter? Your geoboard may help you explore this question.
 - How far can you walk in a day? Don't forget about rest breaks, lunch break, etc.
- 3) Discuss the evaluation criteria with students before they write up their solution. This is a good assignment for collaboration with a language arts teacher for the revising and editing of the writing.

Reference: Danielson, C. (1997). *A Collection of Performance Tasks and Rubrics: Middle School Mathematics*. Larchmont, NY: Eye on Education.



A Country Mile

In an old folktale, a poor peasant is offered as much land as he can walk around from sunup to sundown. If you were given that offer, how much land could you claim? What shape would it be?

Your answer should include these items:

- Drawings of different possible shapes with area and perimeters;
- A clear presentation of your calculation methods
the length of the day
the distance you can walk
the area you claim;
- Clearly labeled work; and
- Neat and accurate writing.



Activity: Similarity and Congruence with Geostrips

Format: Individual or Pairs

Objectives: Participants will create similar and congruent figures using geostrips and compare the figures to determine which properties are the same and which are different.

Related SOL: 6.14, 6.15, 7.9, 7.11

Materials: Geostrips for each student or pair of students, protractors or angle rulers

Time Required: Approximately 30 minutes

Directions:

- 1) Ask the participants to define congruent figures. Guide the discussion towards a definition that includes the fact that all properties of the two figures are exactly the same and that the two figures would match perfectly if laid one on top of the other. Be sure to include proper vocabulary (side, vertex, angle) in this discussion so that the participants are familiar with the terminology as they construct shapes.
- 2) Have participants take any three geostrips and make a triangle. Ask them to compare their triangle with the triangle of a partner who used different strips. What do they notice? Look for answers such as the fact that one side or one angle matches (is congruent) but that not all the parts match. These triangles, then, are not congruent figures.
- 3) Next ask each participant to construct a triangle that is congruent to his/her own triangle. Again, compare the two and notice that each part matches exactly. These triangles are congruent figures.
- 4) Ask participants to construct a triangle using the shortest red, yellow, and blue strips. Participants should then construct a second triangle using the medium length red strip and the longer blue and yellow strips.
- 5) Ask participants to compare the two triangles, noticing what is similar and what is different. They should observe that the angles are congruent but the sides are not. Encourage the participants to use other strips from their supply to determine the 2:1 ratio between the two sets of strips.



- 6) Share with the participants that polygons whose angles are congruent and whose sides are proportional are similar figures. Emphasize that this is a different use of the term similar than we use in everyday English. Students often think similar means “almost alike” even in mathematics. It is important that they realize the term has a precise definition in geometry.
- 7) This activity can be repeated with other polygons.



Activity: Perimeters and Areas of Similar Triangles

Format: Small Group

Objectives: Participants will describe the relationship between area and perimeter of similar triangles in terms of the relationship of the triangle sides.

Related SOL: 6.11, 7.7, 7.11

Materials: A dynamic geometry software program, Data Chart: Area and Perimeter of Similar Triangles.

Time Required: Approximately 45 minutes

- Directions:**
- 1) This exercise assumes participants are familiar with a dynamic geometry software program enough to construct a triangle and measure its area and perimeter.
 - 2) Pose the following question to participants: If two triangles are similar, what is the relationship between their areas? Between their perimeters?
 - 3) Participants can begin thinking about this question by looking at the grid that appears on the Data Chart. Ask participants to name triangles using the marked vertices and to determine if the triangles are similar. When the participants have found two similar triangles, ask them to calculate the area and the perimeter of each triangle.
 - 4) Using a dynamic geometry software program, have participants test their hypothesis on triangles they create. They should use the dilation command within the software to create similar triangles and keep a record of their findings in the chart found in the Data Chart. Emphasize with participants the importance of calculating each ratio in the same way. The side ratio should be Side 1: Side 2, and the area and perimeter ratios should be calculated in the same fashion.
 - 5) If participants do not have access to sketching software, they can create triangles on a geoboard or work from triangles you provide on a handout.
 - 6) As participants discuss their findings, remind them that their examples are supporting evidence, not proof that their hypothesis always holds.



GEOMETRY

When participants teach this lesson, it is appropriate to collaborate with a science teacher and talk about the difference between mathematical proof (where we show that X always holds true) and scientific proof (where the preponderance of the evidence supports the hypothesis and we can generally make accurate predictions) as this will be an important distinction in more advanced mathematics classes.



Data Chart: Area and Perimeter of Similar Triangles

Trial	Side Ratio	Area 1	Area 2	Area Ratio	Perimeter 1	Perimeter 2	Perimeter Ratio



Activity: Making and Using a Hypsometer

Format: Individual or Small Group

Objectives: Participants use the hypsometer and their knowledge of the proportional relationship between similar triangles to determine the height of an object not readily measured directly.

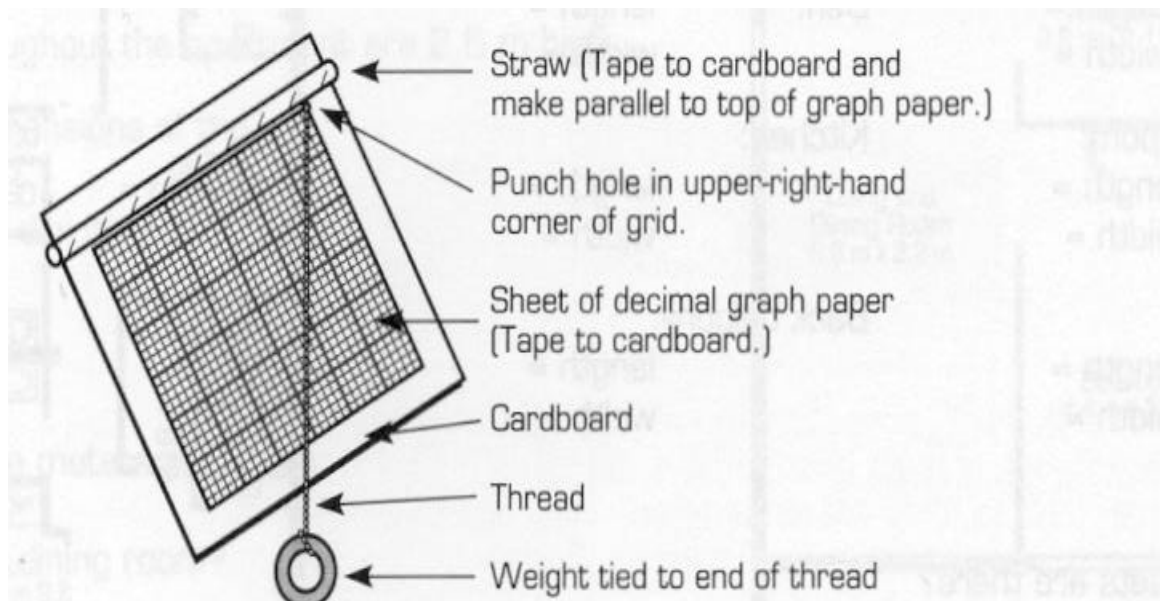
Related SOL: 6.15, 7.11

Materials: For each hypsometer you need a straw, decimal graph paper, cardboard, thread, a small weight, tape, a hole punch, scissors, and a meter stick.

Time Required: Approximately 90 minutes

Directions: To make the hypsometer:

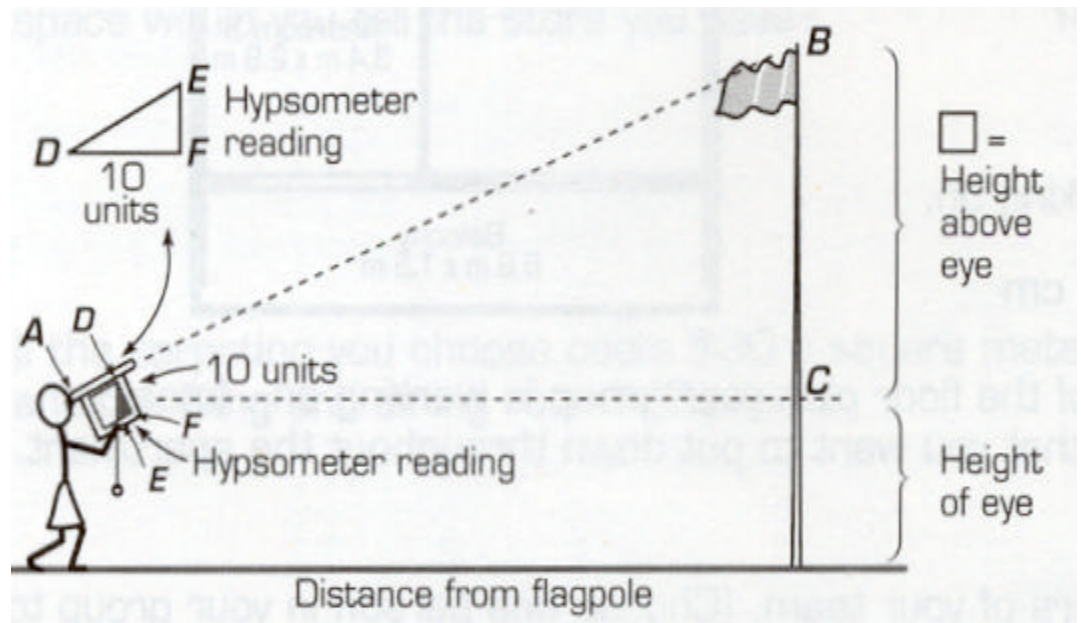
- 1) Tape a sheet of decimal graph paper to a piece of cardboard.
- 2) Tape the straw to the cardboard so that it is parallel to the top of the graph paper.
- 3) Punch a hole in the upper right corner of the grid. Pass one end of the thread through the hole and tape it to the back of the cardboard. Tie the weight to the other end of the thread.





To use the hypsometer:

- 4) Use a meter stick to measure the height of your eye and the distance to the object you wish to measure.
- 5) Look through the straw at the top of the object you wish to measure. Use your finger to hold the string where it hangs so you can record the hypsometer reading. Be careful not to move your finger when you take the hypsometer from your eye to read the measurement!
- 6) To find the height of the flagpole, recognize that triangles ABC and DEF are similar. Thus, BC can be found using the following ratio — $AC:DF::BC:EF$.



Reference:

NCTM Addenda Series, *Measurement in the Middle Grades*



Activity: Human Circle

Format: Large Group

Objective: Participants will develop a definition of a circle and a sphere after creating a human circle.

Related SOL: 6.12

Materials: Enough pieces of string of equal length, for each participant but one; chalk (optional)

Time required: Approximately 15 minutes

Directions:

- 1) Clear a space larger than twice the length of the string cut or go outside to the playground or parking lot.
- 2) Choose one participant to be the center of the circle.
- 3) Have this center person hold the ends of all the strings in one hand, making a fist with all the strings coming out of the top. This person should crouch down, with their arm held over their head.
- 4) Have every other person take an end of string and back up so the string is taut, spacing themselves around the center person in all directions.
- 5) (Optional) Draw a circle with chalk on the ground or floor to approximate the circle created by the humans.
- 6) Discuss the circle as the set of all points in a plane that are the same distance from the center.
- 7) Extend the idea to the set of all points in space that are the same distance from a center point. Contrast a circle and a sphere.



Middle School Geometry Session 2

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Spatial Relationships	Square It	52	6.14, 7.9	Square It	Playing board, grid paper, colored one-inch square markers
	Pick Up the Toothpicks	54	6.14	Pick Up the Toothpicks	Toothpicks
	Partition the Square	56	6.14, 6.15, 7.9	Partition the Square	Paper
Tangrams	Make Your Own Tangrams	59	6.13, 6.14, 6.15, 7.9, 7.11	Directions for Making Tangrams	Paper, scissors
	Area and Perimeter Problems/Tangrams	61	6.11, 7.7	Area and Perimeter with Tangrams	Tangram set
	Spatial Problem Solving with Tangrams	64	6.14, 6.15, 7.9	Problem Solving with Tangrams Sheets 1, 2	Tangram set, puzzles
Soma Cubes	Constructing the Soma Pieces	68	6.17, 7.8, 8.7, 8.8, 8.9	Instructor Reference Sheet, Soma Views, Soma Pieces: Surface Area and Volume	27 wooden cubes, sugar cubes, or snap cubes/participant, permanent markers
	Building the Soma Cube	73	6.17, 7.8, 8.7, 8.8, 8.9	Build This Cube, Soma Solutions Recording Sheet	7 Soma pieces from previous lesson
	Making 2-D Drawings of 3-D Figures	77	6.17, 8.9	Isometric Dot Paper	7 Soma pieces from previous lesson
	Cube Structures	79	6.17, 8.9		One-inch cubes



Topic: Spatial Relationships

Description: To build spatial visualization skills, students need a wide variety of experiences, including building and dissecting figures from different perspectives. In these activities, participants will explore spatial relationships by playing a strategy game with squares, solving a toothpick triangle puzzle, partitioning squares into smaller squares, and using square dissection puzzles.

Related SOL: 6.14, 6.15, 7.9



Activity: Square It

Format: Small Group

Objectives: Participants will recognize squares and gain practice in visualization. As an extension, students will determine the area of a square by counting the number of square units needed to cover it.

Related SOL: 6.14, 7.9

Materials: Playing board, Square It Activity Sheet or 8 x 11 one-inch grid paper, and colored one-inch square markers of two different colors.

Time Required: Approximately 15 minutes

Directions:

- 1) Participants play this game in pairs. Players choose who starts.
- 2) Player places a marker of his or her color on a vacant box on the playing board. Players alternate placing markers.
- 3) The winner is the player to first recognize a SQUARE on the board where all four corners are his or her color. Players check for "squareness" by counting the lengths of the sides. Winning squares may range from 2 x 2 to 7 x 7.
- 4) (Optional) The winning player must state the area of the winning square.



Square It



Activity: Pick Up the Toothpicks

Format: Small Group

Objectives: Participants will recognize triangles and gain practice in spatial visualization.

Related SOL: 6.14

Materials: 11 toothpicks per participant, Pick Up The Toothpicks Activity Sheet

Time Required: Approximately 10 minutes

- Directions:**
- 1) Pass out 11 toothpicks per participant. Tell them to use the toothpicks as they work through the problems posed.
 - 2) Discuss the directions on the Activity Sheet. Eleven toothpicks are arranged as shown to give five triangles. For each problem, begin with the original 11-stick configuration. Then:
 - A. remove two toothpicks and show **three** triangles;
 - B. remove one toothpick and show **four** triangles;
 - C. remove three toothpicks and show **three** triangles; and
 - D. remove two toothpicks and show **four** triangles.



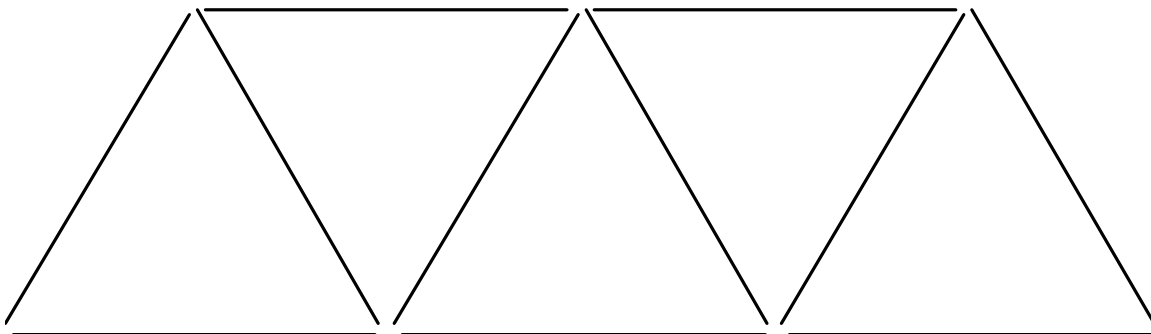
- 3) Be sure to discuss the fact that all the sides don't have to be the same length in order for a three-sided figure to be a triangle.



Pick Up the Toothpicks

Eleven toothpicks are arranged as shown to give five triangles. For each problem, begin with the original 11-stick configuration. Then:

- A. remove two toothpicks and show **three** triangles;
- B. remove one toothpick and show **four** triangles;
- C. remove three toothpicks and show **three** triangles;
and
- D. remove two toothpicks and show **four** triangles.





Activity: Partition the Square

Format: Individual /Large Group

Objectives: Participants will partition squares into smaller squares and will explain how they know that the smaller figures are really squares.

Related SOL: 6.14, 6.15, 7.9

Materials: Partition the Square Activity Sheet, scratch paper

Time Required: Approximately 30 minutes

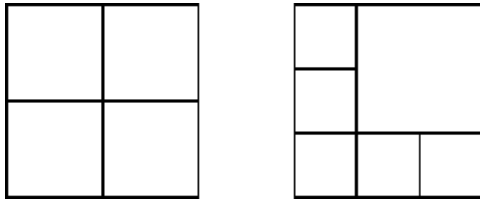
Directions:

- 1) Distribute the Partition the Square Activity Sheet. Explain to the participants that they are to divide each square into smaller squares and that there are many ways to determine each number of squares. Use the two sample partitions to point out that the squares don't have to be the same size, just that four sides of each square must be congruent, and that overlaps will not count.
- 2) Circulate around the room, referring participants to the two samples if they need assistance. Also, look for non-square rectangles and remind the participants that all four sides of a square are congruent.
- 3) After they have had a few minutes to work, ask the participants to share their solutions. Start out with labels 7 to 15 and ask for a volunteer to do each one. Ask participants to add their method if they have a different way of partitioning than the one shown.
- 4) After all solutions have been shared by the participants, challenge the group to justify that each really is composed of squares. Tell them that you will allow them to assume that angles that look like right angles are right angles. Perhaps the simplest way of justifying four congruent sides in each of the drawn squares is to think of the original square as a unit square and then label the sides accordingly.

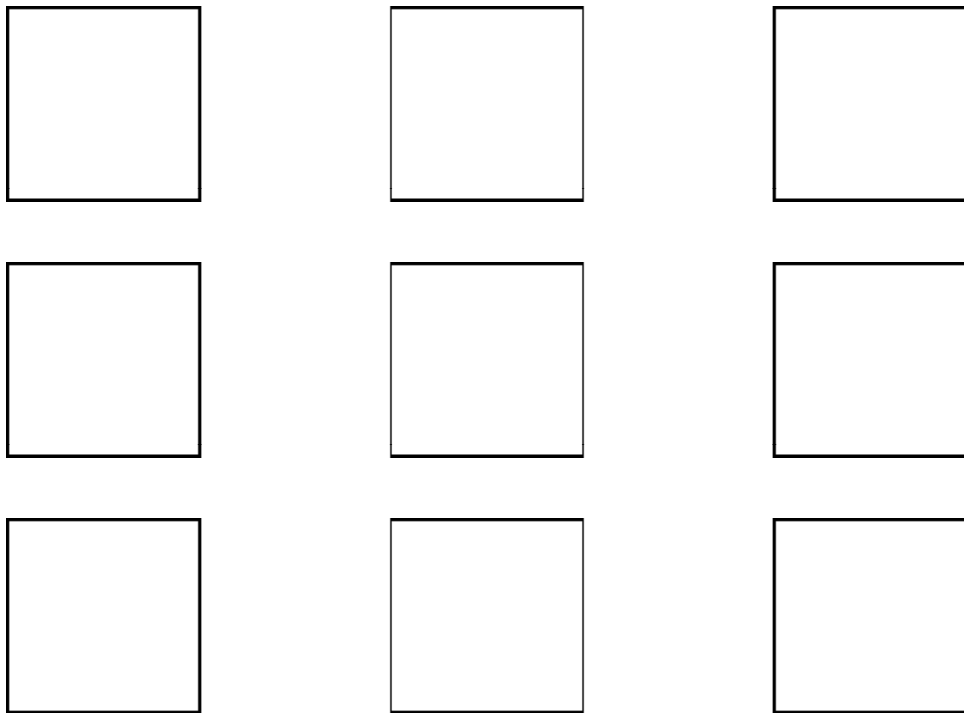


Partition the Square

A square can be partitioned into squares in more than one way. Shown below are squares partitioned into 4 smaller squares and 6 smaller squares.



Use these partitioning ideas to find ways to partition the nine squares below into 7 to 15 smaller squares.





Topic:

Tangrams

Description:

Participants will make their own tangrams and will use them to explore area and perimeter relationships in geometric figures. They will engage in problem-solving puzzles using tangrams.

Related SOL:

6.11, 6.13, 6.14, 6.15, 7.7, 7.9



Activity: Make Your Own Tangrams

Format: Small Group

Objectives: Participants will construct their own tangrams and identify properties of the seven tangram pieces.

Related SOL: 6.13, 6.14, 6.15, 7.9, 7.11

Materials: Paper suitable for folding such as copy paper (one sheet per student), scissors, one set of overhead tangrams, Directions for Making Tangrams Activity Sheet

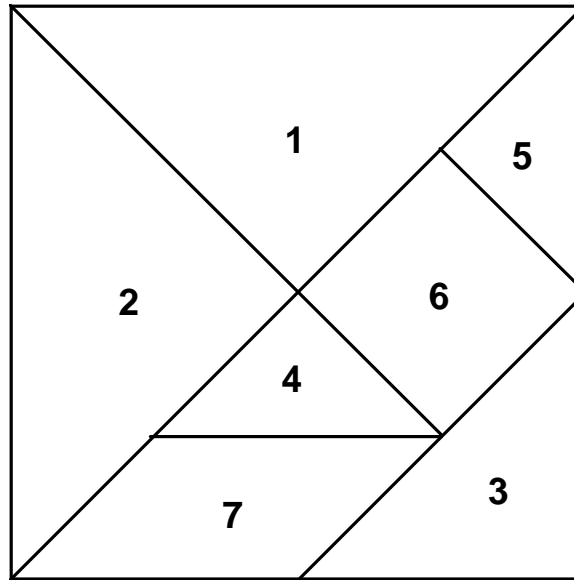
Time Required: 30 minutes

Directions:

- 1) Distribute directions for making a set of the seven tangram pieces (Directions for Making Tangrams Activity Sheet) or give the directions orally for participants to follow individually as you make a master set as a demonstration.
- 2) Ask participants to make a square out of all seven tangram pieces.
- 3) Have participants label the pieces by number as indicated in the diagram below. They should also identify each by the name of the figure. Put a set of overhead tangrams on the overhead projector and discuss:
 - Identify each tangram piece by the name of the figure.
 - Which figures are congruent? How do you know?
 - Which triangles are similar? How do you know? Can you write a proportion to express the relationship between the lengths of the triangle sides?
- 4) Have participants find the measure of each angle of each figure. They should trace each figure and label the angle measures.



Directions for Making Tangrams



- 1) Fold the lower right corner to the upper left corner along the diagonal. Crease sharply. Cut along the diagonal.
- 2) Fold the upper triangle formed in half, bisecting the right angle, to form Piece 1 and Piece 2. Crease and cut along this fold. Label these two triangles "1" and "2."
- 3) Connect the midpoint of the bottom side of the original square to the midpoint of the right side of the original square. Crease sharply along this line and cut. Label the triangle "3."
- 4) Fold the remaining trapezoid in half, matching the short sides. Cut along this fold.
- 5) Take the lower trapezoid you just made and connect the midpoint of the longest side to the vertex of the right angle opposite it. Fold and cut along this line. Label the small triangle "4" and the remaining parallelogram "7."
- 6) Take the upper trapezoid you made in Step 4. Connect the midpoint of the longest side to the vertex of the obtuse angle opposite it. Fold and cut along this line. Label the small triangle "5" and the square "6."



Activity: Area and Perimeter Problems with Tangrams

Format: Individual/Small group

Objectives: Participants will explore area relationships with tangrams.

Related SOL: 6.11, 7.7

Materials: A set of tangrams for each participant, Area and Perimeter with Tangrams Activity Sheet

Time Required: 30 - 40 minutes

Directions:

- 1) Give participants the Area and Perimeter with Tangrams Activity Sheet. Make sure they also have a set of tangrams. Ask them to work alone or in small groups to complete the tasks outlined on the Activity Sheet. Circulate around the room, helping participants who need assistance.
- 2) Invite participants to the overhead projector to describe how they found the answers.



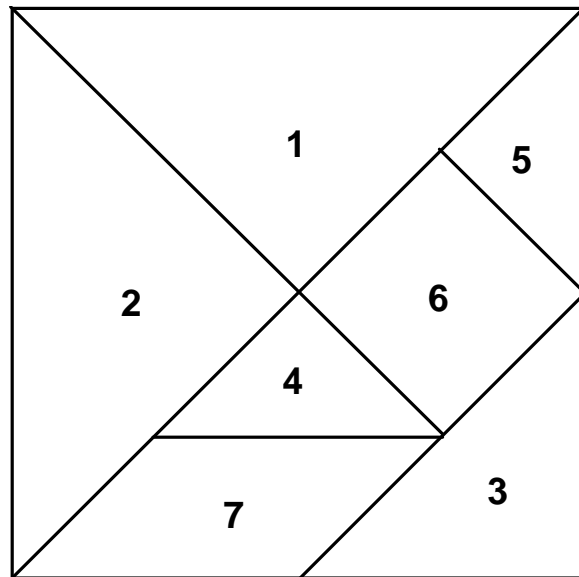
Area and Perimeter With Tangrams

- 1) If the area of the composite square (all seven pieces -- see below) is one unit, find the area of each of the separate pieces in terms of the area of the composite square.

Piece #	area
1	
2	
3	
4	
5	
6	
7	

- 2) If the smallest triangle (piece #4 or #5) is the unit for area, find the area of each of the separate pieces in terms of that triangle.

Piece #	area
1	
2	
3	
4	
5	
6	
7	





GEOMETRY

- 3) If the smallest square (piece #6) is the unit for area, find the area of each of the separate pieces in terms of that square. Enter your findings in the table below.
- 4.) If the side of the small square (piece #6) is the unit of length, find the perimeter of each piece and enter your findings in the table.

piece #	area	perimeter
1		
2		
3		
4		
5		
6		
7		









- Activity:** Spatial Problem Solving with Tangrams
- Format:** Independent/Small Group
- Objectives:** Participants will create geometric figures with the tangram pieces.
- Related SOL:** 6.14, 6.15, 7.9
- Materials:** A set of tangrams for each student; Spatial Problem Solving with Tangrams Activity Sheet, Tangram Puzzles Activity Sheet
- Time Required:** Variable, allow 30 minutes to get started. Participants may work independently over a period of a week or so and turn in solutions at a later session.
- Directions:** Distribute Activity Sheets and have participants work individually or in small groups to solve the tangram puzzles.



Spatial Problem Solving with Tangrams

Use the number of pieces in the first column to form each of the geometric figures that appear in the top of the table. Make a sketch of your solution(s). Some have more than one solution while some have no solution.

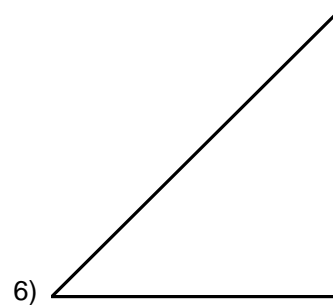
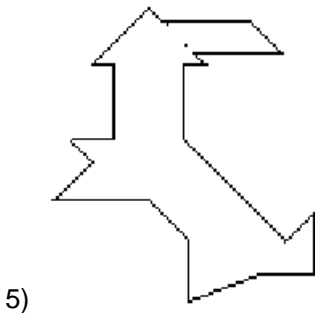
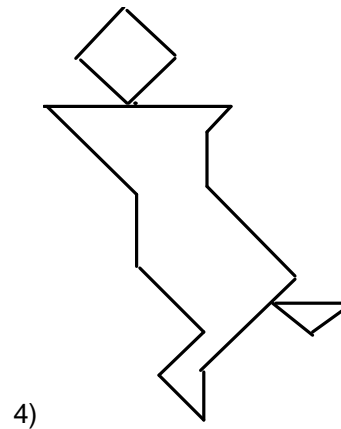
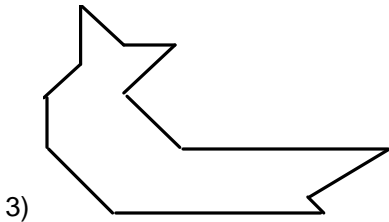
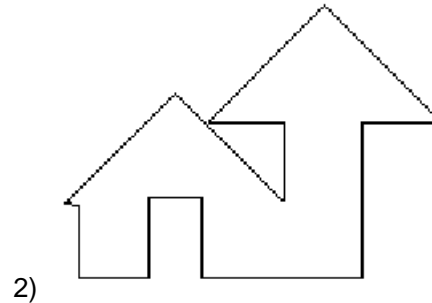
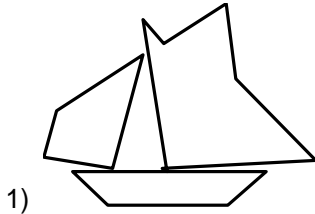
Make These Polygons

Use this many pieces	 Square	 Rectangle	 Triangle	 Trapezoid	 Trapezoid	 Parallel-ogram
2						
3						
4						
5						
6						
7						



Tangram Puzzles

Can you make these figures using all seven tangram pieces? Make a sketch of your solutions.



Design your own tangram picture. Trace the outline and give it a name. Submit the outline and a solution key.



Topic: **Spatial Relations Using the Soma Cube**

Description: Participants will construct the seven Soma pieces and solve problems involving the pieces.

Related SOL: 6.17, 7.8, 8.7, 8.8, 8.9



Activity: **Constructing the Soma Pieces**

Format: Whole group directions and discussion followed by individual problem-solving.

Objectives: Participants will solve spatial problems and will find surface area and volume of constructed figures.

Related SOL: 6.17, 7.8, 8.7, 8.8, 8.9

Materials: 27 wooden cubes for each student (or sugar cubes, or Snapcubes), glue, permanent markers; glue, and markers; Instructor Reference Sheet; Soma Views from Top, Front, and Side Activity Sheet; Surface Area and Volume of the Soma Pieces Activity Sheet

Time Required: 45 minutes

Directions: 1) Give out 27 wooden 1-inch cubes or 27 snapcubes to each participant or small group of participants if materials are limited. Present them with the following problem:

- How many different ways can you join three cubes face-to-face? These are called “tricubes.”

Have participants try finding them with the cubes and discuss the results. Tell them that if a tricube can be flipped or repositioned (reflected, rotated) in such a way that it is exactly like a tricube already made, then it is not different from the other one.

There are only two different tricubes. However, for this activity we only need to save the non-rectangular one. Have participants put aside the rectangular one (every face is a rectangle!).

- 2) Have participants find all possible non-rectangular tetracubes (4 unit cubes joined face-to-face). Discuss what they find. There should be six different ones (see reference page that accompanies this lesson). You may have participants glue the wooden cubes together (or snap the snapcubes together) and number the completed pieces according to the reference page. Discuss the nature of the pieces:
 - Are any of the pieces reflections of each other? (If you put a mirror next to one piece, will you see the other in the mirror?)

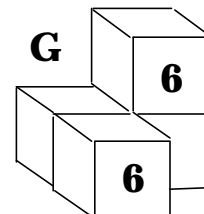
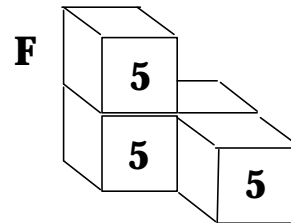
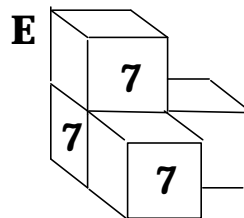
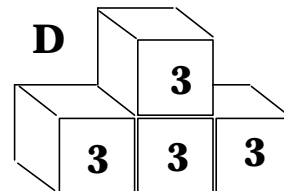
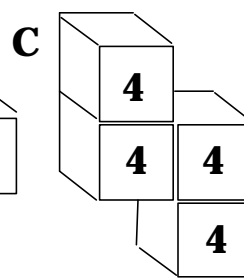
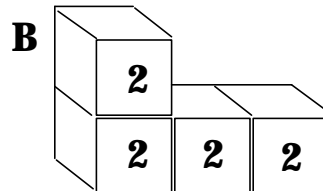
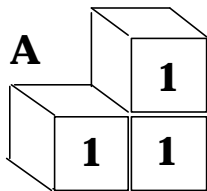


- Which pieces can be placed so that they are only one unit high? (Pieces #1, #2, #3, #4)
 - Which pieces must occupy space that is 2 units high? (Pieces #5, #6, #7)
 - Which of the pieces have a line of symmetry on a given face?
- 3) Review the concept of volume by identifying one of the wooden cubes or one of the snapcubes as the unit. Discuss the face of the unit cube as the unit of area. Have participants find the surface area and volume of each of the numbered solids #1 - #7 and complete the table in Handout 2.7 Surface Area and Volume Activity Sheet. Discuss their findings:
- Did the pieces with the same volume have the same surface area?
 - Did the pieces with the same surface area have the same volume?
- 4) Give participants diagrams of the top, side, and bottom view of Soma pieces in Soma Views from the Top, Front, and Side Activity Sheet and have them identify the pieces from these views. Have them draw the diagrams for the remaining pieces.

Extension: Ask participants to build all “pentacubes” that can be made with five cubes each.

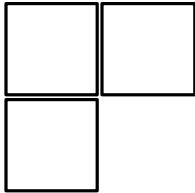


Instructor Reference Sheet

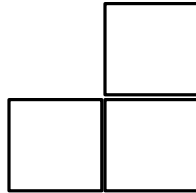




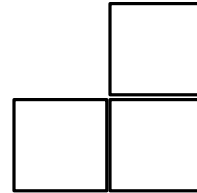
Soma Views from Top, Front, and Side



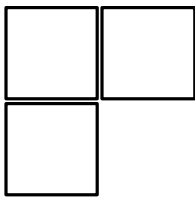
top



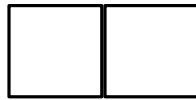
front



right side



top



front



right side



Surface Area and Volume of the Soma Pieces

piece #	surface	
	area	volume
1		
2		
3		
4		
5		
6		
7		



Activity: **Building the Soma Cube and other Structures with Soma Pieces**

Format: Small group

Objectives: Participants will use the seven Soma pieces to build the 3x3x3 cube and other structures that use all seven pieces.

Related SOL: 6.17, 7.8, 8.7, 8.8, 8.9

Materials: Seven Soma pieces from previous lesson, Build This Cube and Soma Solutions Recording Sheet Activity Sheets

Time Required: 45 minutes; some participants may want to extend the activities independently

Directions: 1) Give participants some history of the Soma cube. It was invented by Piet Hein in Denmark. He was listening to a lecture on quantum physics when the speaker talked about slicing up space into cubes. Hein then thought about all the irregular shapes that could be formed by combining no more than four cubes, all the same size and joined at their faces. In his head he figured out what these would be and that it would take 27 cubes to build them all. From there he showed that the pieces could form a 3x3x3 cube.

2) Tell participants: *So now we know that the seven pieces fit together to form a 3x3x3 cube. In fact, there are 1,105,920 different ways to assemble the cube. Try to find one.*

Let participants work until they get a solution.

3) Ask participants how they might make a record of their solution before they take the cube apart. You may show them one way by sharing the solution-recording sheet that accompanies this lesson. It requires that the numbers of the unit cubes be recorded in three layers: top, middle, and bottom. Have participants record their solutions. Note that some will need help in making the correct correspondence of the numbers to the grid. Then have them try to find a different solution and record it.

4) Participants may want to explore the Soma Cube further by going to the web site <http://web.inter.nl.net/users/C.Eggermont/Puzzels/Soma/>



Discuss:

- How many dimensions are represented in the Soma cube?
- How many dimensions are represented in the solution sheet?
- Does anyone want to share any strategies that might help in transferring the 3-dimensional information onto the 2-dimensional representation?

- 4) Now reverse the order of the task. Give participants a written Soma solution and ask them to build that cube. The Build This Cube Activity Sheet contains this example:

Top			Middle			Bottom		
6	6	2	6	3	4	3	3	3
5	6	2	5	4	4	7	1	1
5	5	2	7	4	2	7	7	1

- 5) Participants use the Activity Sheet with pictures of structures that can be built with the seven Soma pieces. Once they have succeeded at building any of the structures, they should label the drawings with the numbers of the appropriate pieces.
- 6) Have participants complete a table in which they compare the volume of the structures and the surface area. Discuss their observations with the whole class:
- What is the volume of the completed Soma cube?
 - What is the volume of the other structures you built?
 - How can you tell the volume of the structures by only studying the picture and not actually building them?
 - How can you figure out the surface area of the structures without actually building the figures?



Build This Cube

Top

6	6	2
5	6	2
5	5	2

Middle

6	3	4
5	4	4
7	4	2

Bottom

3	3	3
7	1	1
7	7	1



Soma Solutions Recording Sheet

Top	Middle	Bottom

Top	Middle	Bottom

Top	Middle	Bottom



Activity: Making 2-Dimensional Drawings of 3-Dimensional Figures

Format: Participants will work in small groups to apply the drawing techniques that have been demonstrated by the instructor.

Objectives: Participants will use isometric dot paper to make drawings of Soma pieces to make a 2-dimensional drawing of a 3-dimensional figure.

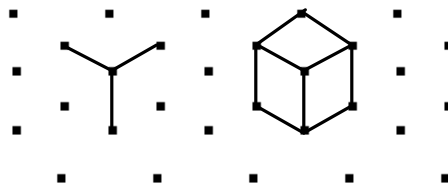
Related SOL: 6.17, 8.9

Materials: Isometric Dot Paper and Soma pieces from previous lesson

Time Required: 45 minutes

Directions:

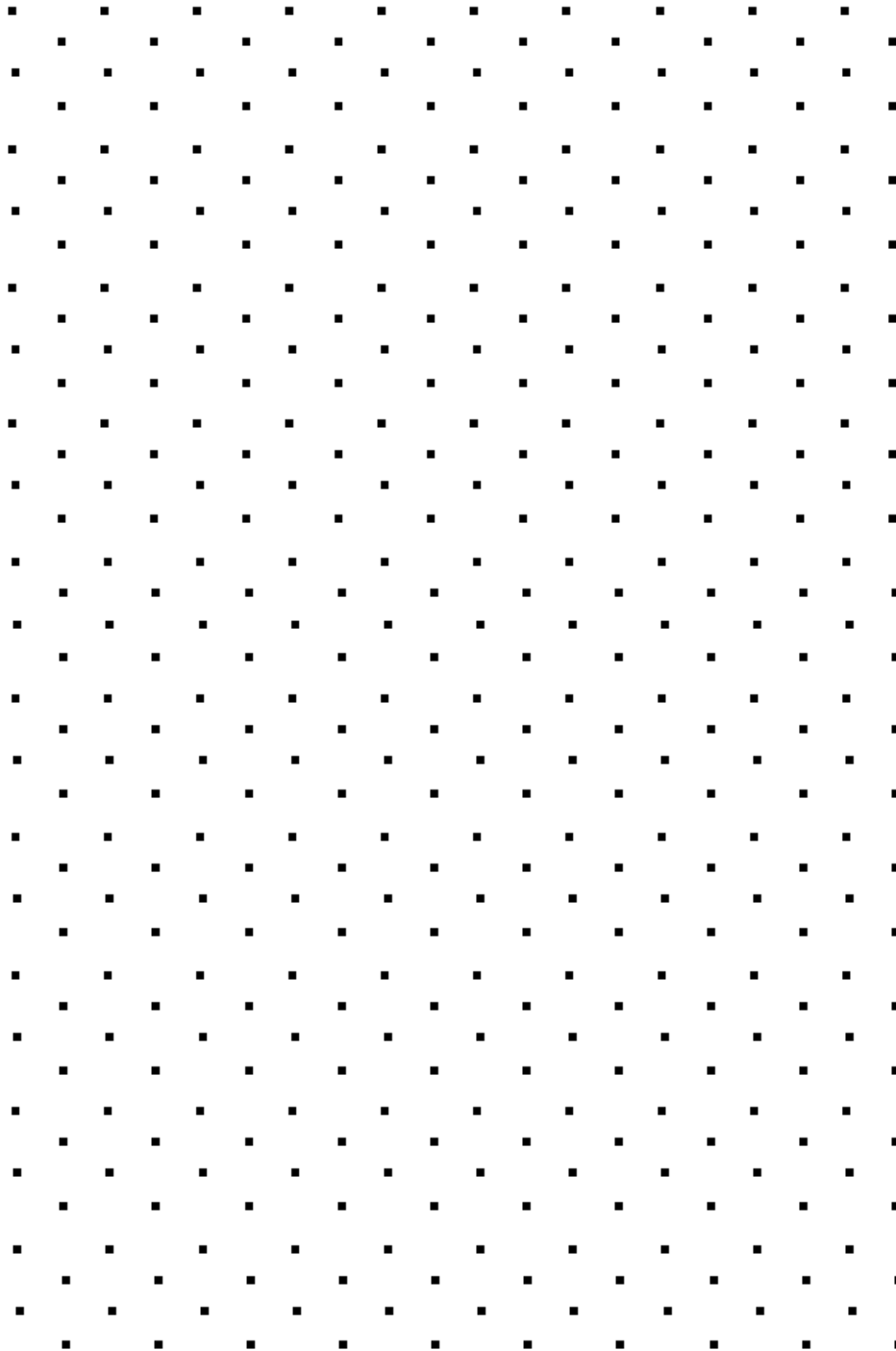
- 1) Distribute the isometric dot paper to participants. Have them study it and discuss how it is different from regular graph paper. Discuss:
 - What does the prefix “iso” mean?
 - How does this apply to the way the paper is designed?
- 2) Have participants practice drawing single cubes while working in small groups so they can help each other. (Hint: the easiest way to show this is by drawing a Y in the center and then circumscribing a hexagon around it-- below.)



- 3) Show participants how to position one of the seven Soma pieces on a diagonal so that they can draw it on the isometric paper. Draw one on the overhead while talking through the process.
- 4) Have participants work in groups to draw all seven Soma pieces on isometric dot paper. Each participant should complete his or her own drawings with the help of group members.
- 5) Challenge participants to design a structure using all seven Soma pieces and draw it on the isometric dot paper.



Isometric Dot Paper





Activity: **Cube Structures**

Format: Small group

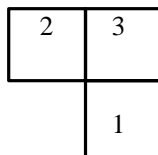
Objectives: Participants will draw the top, front, and side views of cube structures and will build structures from drawings of the three views.

Related SOL: 6.17, 8.9

Materials: One-inch cubes and paper

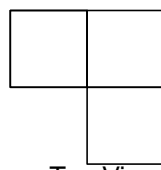
Time Required: 20 minutes

Directions: 1) Give each participant 10 one-inch cubes. Ask them to build a structure that stacks six cubes as indicated below (this is the view from the top).

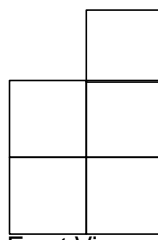


2) Ask participants to draw the top view, the front view, and the right-side view of the structure. Discuss and check for accuracy.

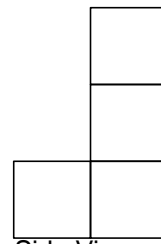
Key:



Top View



Front View



Side View

3) Have participants work in pairs with a barrier to block the view of the partner's structure. Each person should build a structure with some of the blocks and then draw the three views -- top, front, and side.



GEOMETRY

- 4) Each person should then pass the drawings to the partner. The partners build the structures according to the pictures. Remove the barrier to check the accuracy of the structure.
- 5) Repeat the procedure with a new structure.



Middle School Geometry Session 3

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Transformational Geometry: Tessellations	Sums of the Measures of Angles of Triangles	84	6.13, 6.14, 8.6		Paper, scissors, and rulers
	Do Congruent Triangles Tessellate?	87	6.14, 6.15, 8.8	Types of Triangles, Sum of the Measures of the Angles, Tessellations of Triangles 1 and 2, Congruent Scalene Triangles	Paper, scissors, and rulers
	Do Congruent Quadrilaterals Tessellate?	94	6.14, 6.15, 7.9, 8.8	Sum of the Measures of the Angles, Tessellations of Quadrilaterals 1, 2, and 3	Paper, scissors, and rulers
	Tessellations by Translation	99	8.8	Tessellations by Translation, Square	1 large square piece of paper per participant, rulers and scissors
	Tessellations by Rotation	102	8.8	Tessellations by Rotation	1 large square piece of paper per participant, rulers and scissors



Topic: Transformational Geometry: Tessellations

Description: Patterns of geometric design are all around us. We see them every day, woven into the fabric of the clothes we wear, laid underfoot in the hallways of the buildings where we work, and printed on the wallpaper of our homes. Whether simple or intricate, such patterns are intriguing to the eye. We will explore a special class of geometric patterns called *tessellations*. Our investigation will interweave concepts basic to art, to geometry, and to design.

The word *tessellation* means, "a design that completely covers the surface with a pattern of figures with no gaps and no overlapping." It comes to us from the Latin *tessela*, which was the small, square stone, or tile used in ancient Roman mosaics. *Tilings* and *mosaics* are common synonyms for tessellations. Much like a Roman mosaic, a *plane tessellation* is a pattern made up of one or more figures, completely covering a surface without any gaps or overlaps. Note that both two-dimensional and three-dimensional figures will tessellate. Two-dimensional figures may tessellate a plane surface, while three-dimensional figures may tessellate space. We will use the word *tessellation* alone to always mean a plane tessellation.

Although the mathematics of tiling can become quite complex, the beauty and order of tessellation is accessible to anyone who is interested. To analyze tessellating patterns, you have to understand a few things about geometric figures and their properties - but all you need to know is easily explained in a few pages.

We will approach this subject through directed exploration. We will be exploring the following questions: Which figures will tessellate (that is, tile a plane without overlapping or leaving spaces)? Why will certain figures tessellate and others not? How many different tessellating patterns can we create using two or more regular, two-dimensional plane figures? Do tessellating designs have symmetry? If so, what kind? How can we use transformations (slides, flips, and turns) to create unique tessellations? What other techniques could we use to generate the intricate designs?



GEOMETRY

In the twentieth century, a number of fine artists have applied the concept of tessellating patterns in their work. The best known of these is Dutch artist, M. C. Escher. Inspired by the Moorish mosaic designs he saw during a visit to the Alhambra in Spain in the 1930s, Escher spent most of his life creating tessellations in the medium of woodcuts. He altered geometric tessellating figures into such forms as birds, reptiles, fish, and people.

Related SOL: 6.13, 6.14, 6.15, 7.9, 8.6, 8.8



Activity: Sum of the Measures of Angles of a Triangle

Format: Large Group/Small group

Objective: Participants will demonstrate that the sum of the measures of the interior angles of any triangle equals 180° .

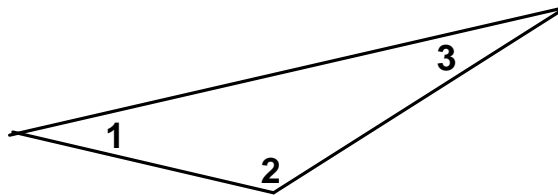
Related SOL: 6.13, 6.14, 8.6

Materials: Paper, scissors, straightedges

Time required: Approximately 15 minutes

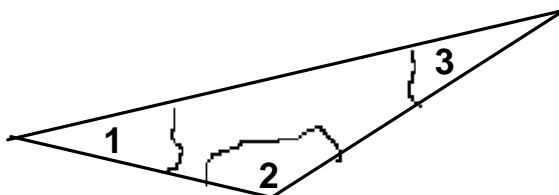
Directions:

- 1) Distribute the paper, scissors, and straightedges.
- 2) Have each participant draw 3 large triangles using a straightedge, and then cut the 3 triangles out. They should label the vertices of one triangle as 1, 2, and 3; label the vertices of the second triangles as 1^* , 2^* , and 3^* ; and label the vertices of the third triangle as $1'$, $2'$, and $3'$.





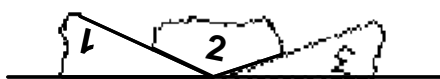
- 3) Have them tear off the corners. (That's right. Tear, not cut. By tearing, you can still determine which was the vertex. It will be the cut part.)



- 4) The participants should draw a dot on the page and a straight line through the dot. Have them place the cut vertex of $\angle 1$ on the dot and the cut side on the line. They can trace the angle or tape it in place. They should choose another angle from the same figure and place its cut vertex on the dot, lining up the side of the first angle not on the line with a side of the second angle, and trace or tape it in place.



- 5) Have them repeat this sequence of steps until all the angles from one figure are adjacent to one another.



- 6) What kind of angle do these combined corners form?
- 7) Repeat this procedure with the other two triangles. Ask if the same thing happens with all three triangles.
- 8) Have the groups share their results and then form conclusions focused on the following issues:
- Do the angles from all of their triangles always equal the same amount?
 - Examine any triangle that turned out differently. Why do you think it did?
 - Based on this activity, what conjecture can you make about the sum of the measure of angles in a triangle?



- d. If you and your group members made 200 triangles, tore off the vertices, lined them up, and they all totaled the same, would this insure that the 201st triangle would come out the same?
- e. If you could find a single triangle that came out differently, you would disprove the conjecture that the sum of the measures of the interior angles in a triangle is always 180° . Any item that doesn't fit your conjecture is called a **counterexample**. A single counterexample is enough to disprove a conjecture. Did you or any of your group members find a triangle whose angle measures didn't add up to 180° ? Unfortunately, not finding one doesn't mean that one doesn't exist. However, it does give us more confidence in our conjecture.



Activity: Do Congruent Triangles Tessellate?

Format: Large Group/Small group

Objective: Participants will cut out sets of congruent triangles of various types and cover the plane with each type of triangle.

Related SOL: 6.14, 6.15, 8.8

Materials: Paper, scissors, rulers, Types of Triangles Activity Sheet, Sum of the Measures of the Angles Activity Sheet, Tessellations of Triangles Activity Sheets 1 and 2, Congruent Scalene Triangles Activity Sheet

Time required: Approximately 20 minutes

- Directions:**
- 1) Remind the participants of the following definition: **Tessellation – a design that completely covers a surface with a pattern of figures with no gaps and no overlapping.**
 - 2) Since a triangle is the simplest two-dimensional plane figure, we will start with the triangle in our investigation of which two-dimensional plane figures tessellate. Also, to keep things simple, have the participants explore tessellating with a single triangular figure rather than combinations of different triangular figures.
 - 3) Review the definition of congruent triangles -- triangles that are the same size and shape.
 - 4) Before proceeding with the investigation, have the participants look at different types of triangles so that we can consider each type separately. Triangles are classified according to the relationships and the size of their sides and angles. Examples of each triangle type are shown on the Types of Triangles Activity Sheet.
 - 5) Ask the participants to explore the following questions:

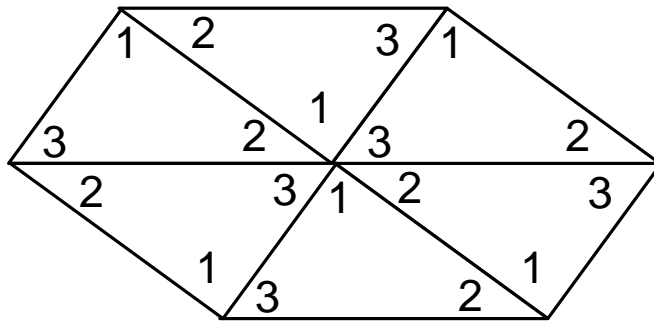
Which triangles (if any) tessellate?

If some triangles tessellate, do they all?



Start the investigation with scalene triangles. Have the participants draw a scalene triangle and copy it several times to make congruent triangles. They should label the congruent triangles so that all angle 1's are congruent, all angle 2's are congruent, and all angle 3's are congruent. The participants should cut the congruent triangles out and move them around to see if they tessellate. If you don't want the participants to create their own scalene triangles, you can just have them cut out several triangles from the Congruent Scalene Triangles Activity Sheet

- 6) Ask the participants if angles 1, 2, and 3 come have a common vertex. What appears to be the sum of the measures of angles 1, 2, and 3?
- 7) Using six congruent scalene triangles, we can completely fill the space around the common vertex of the six triangles. There are no gaps, no overlaps - a criterion for tessellation. Do you think that any scalene triangle will tessellate the plane? Why or why not? Discuss this question, referring to the Sum of the Measures of the Angles Activity Sheet.

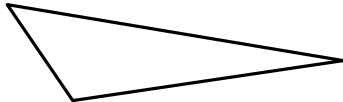


- 8) Try to make a tessellation using six congruent right triangles. Do you think that any right triangle will tessellate the plane? Why or why not?
- 9) Can you make a tessellation using six congruent equilateral triangles or six congruent isosceles triangles?
- 10) Examine the tessellations on the Tessellations of Triangles Activity Sheet. Identify the triangles used to form the tessellations. Do you think that any six congruent triangles can be used to make a tessellation? Why or why not?

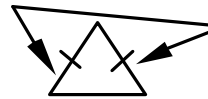


Types of Triangles

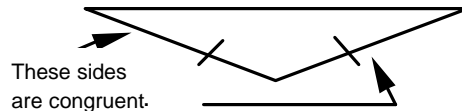
A **scalene** triangle has **no** congruent sides.



An **isosceles** triangle has at least **two congruent sides**.

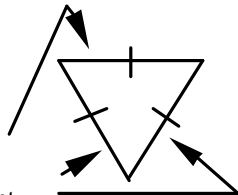


These sides are congruent.



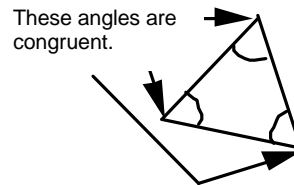
These sides are congruent.

An **equilateral** triangle has three congruent sides.



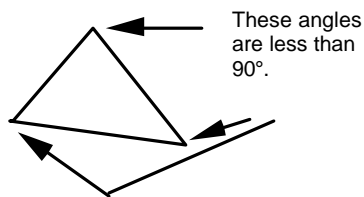
These sides are congruent.

An **equiangular** triangle has all three angles congruent.



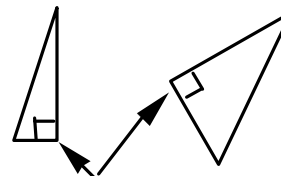
These angles are congruent.

An **acute** triangle has three acute angles.



These angles are less than 90° .

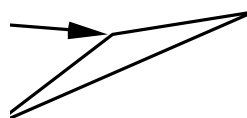
A **right** triangle contains a right angle.



These are 90° angles.

An **obtuse** triangle contains an obtuse angle.

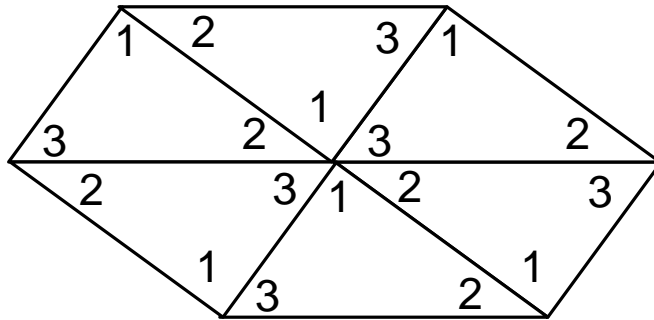
This angle is greater than 90° and less than 180° .





Sum of the Measures of the Angles

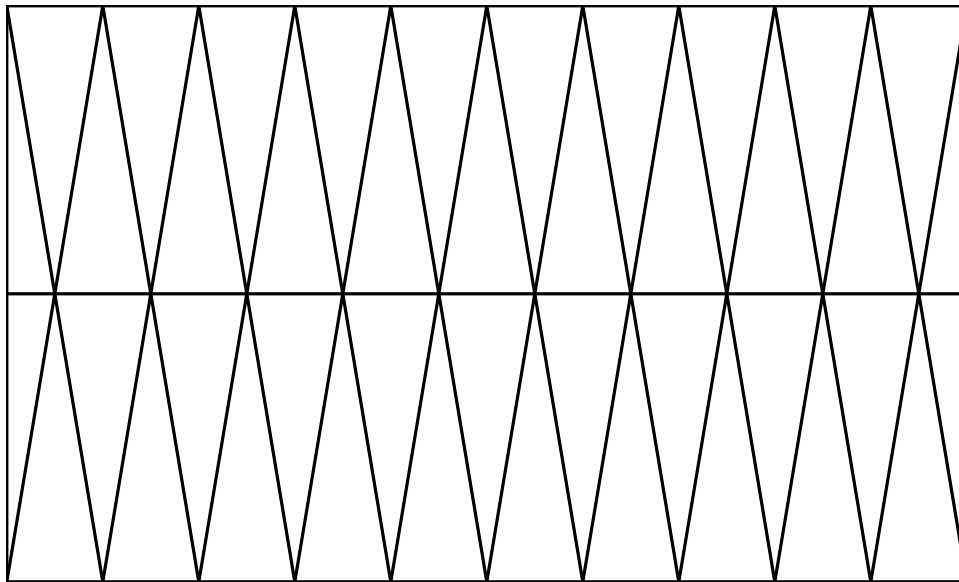
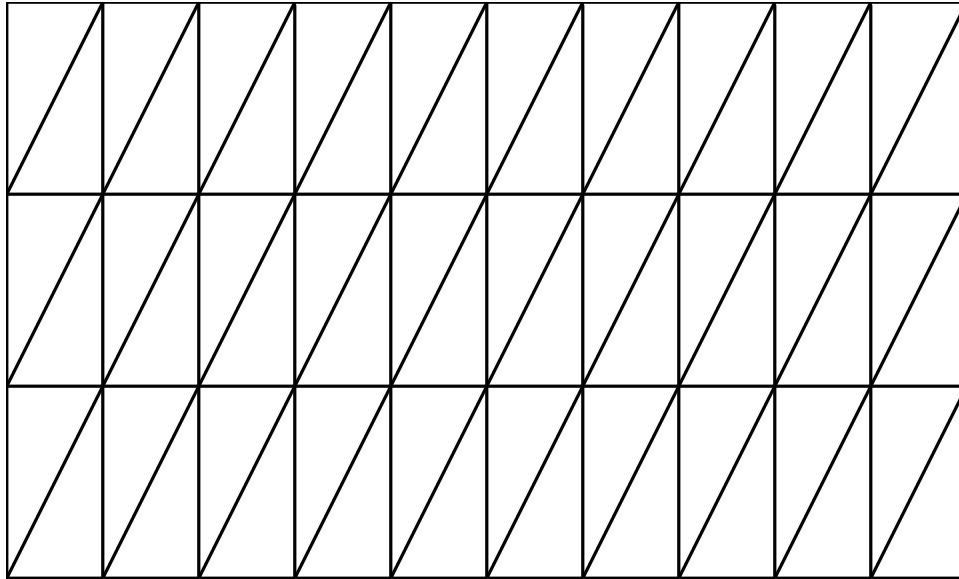
What is the sum of the measures of the angles about the center point?





Tessellations of Triangles

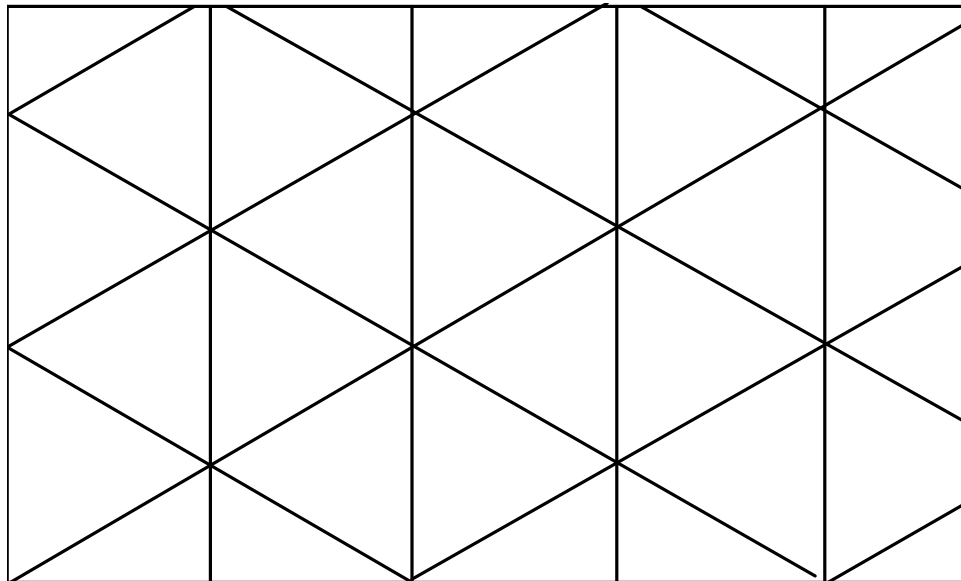
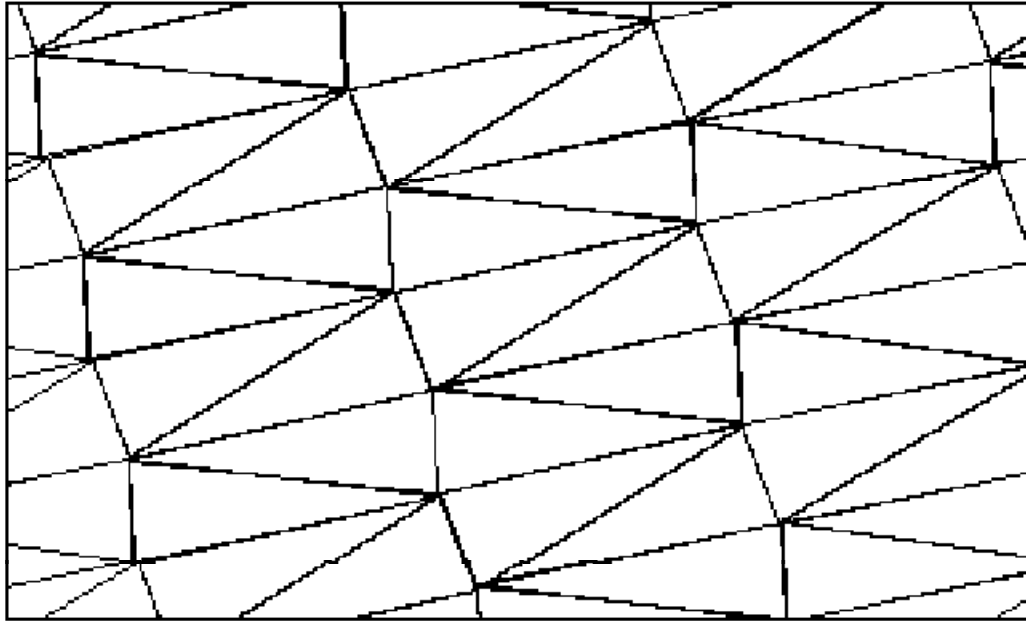
Page 1





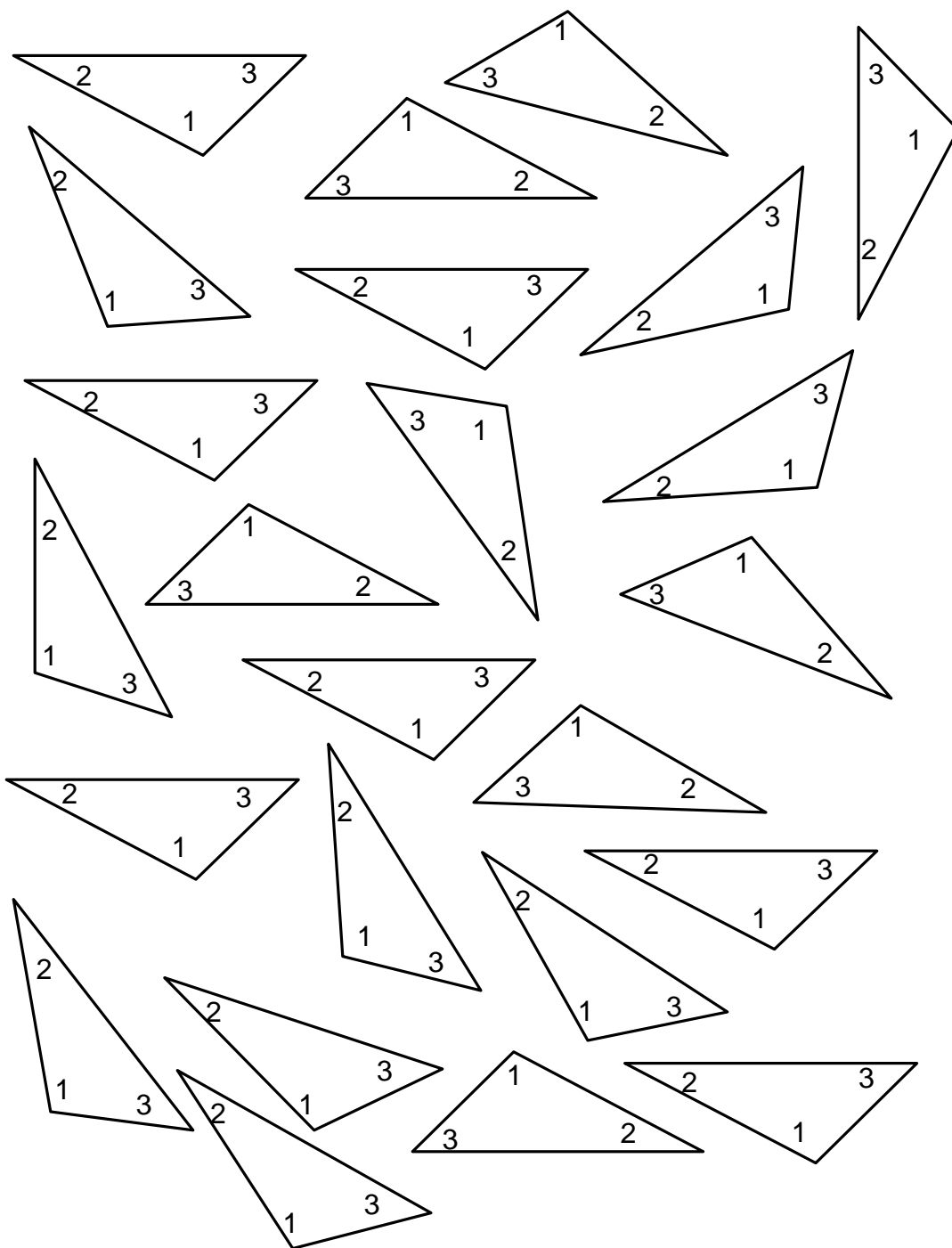
Tessellations of Triangles

Page 2





Congruent Scalene Triangles





Activity: Do Congruent Quadrilaterals Tessellate?

Format: Large Group/Small Group

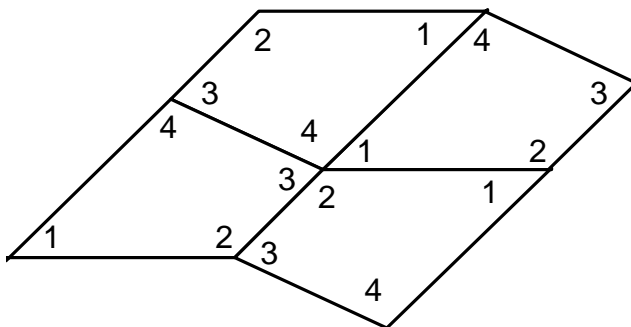
Objective: Participants will determine whether congruent quadrilaterals tessellate.

Related SOL: 6.14, 6.15, 7.9, 8.8

Materials: Paper, scissors, rulers, Sum of the Measures of the Angles Activity Sheet, Tessellations of Quadrilaterals Activity Sheets 1, 2, and 3

Time required: Approximately 15 minutes

- Directions:**
- 1) Repeat the previous activity entitled “Do Congruent Triangles Tessellate?” (page 87), using six or more congruent quadrilaterals (four-sided two-dimensional plane figures). Do your quadrilaterals tessellate?
 - 2) Using four congruent quadrilaterals, we can completely fill all the space around the common vertex point of the four quadrilaterals. There are no gaps, no overlaps - criteria of a tessellation. Do you think that all quadrilaterals will tessellate in the plane? Why or why not? Discuss this question, referring to the Sum of the Measures of the Angles Activity Sheet.

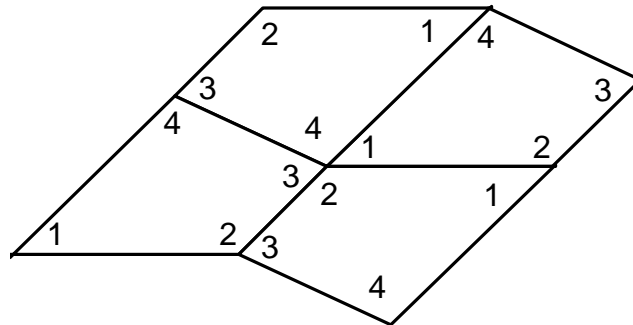


- 3) Examine the tessellations in the Tessellations of Quadrilaterals Activity Sheets. Identify the quadrilaterals used to form the tessellations. Do you think that any four congruent quadrilaterals can be used to make a tessellation? Why or why not?



Sum of the Measures of the Angles

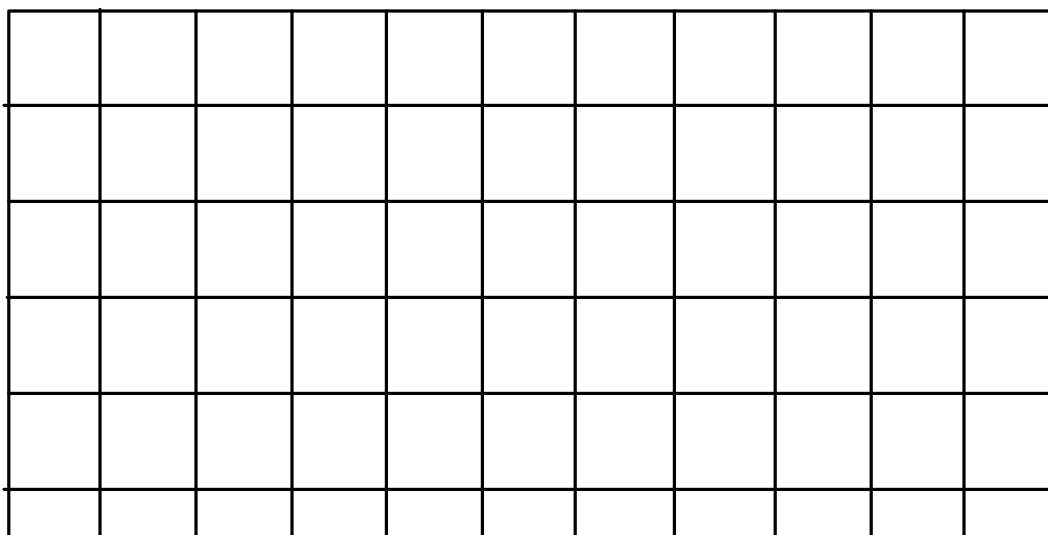
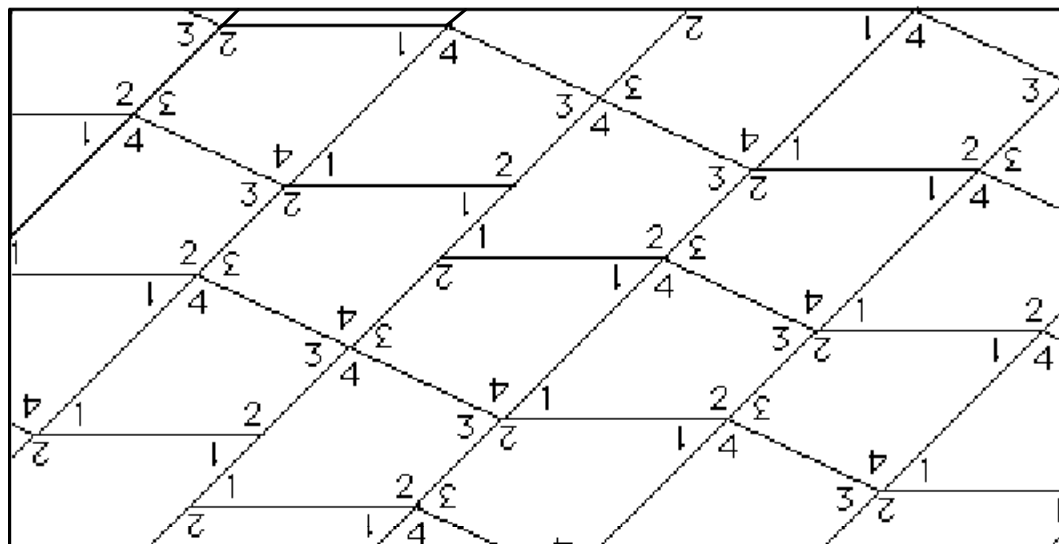
What is the sum of the measures of the angles about the center point?





Tessellations of Quadrilaterals

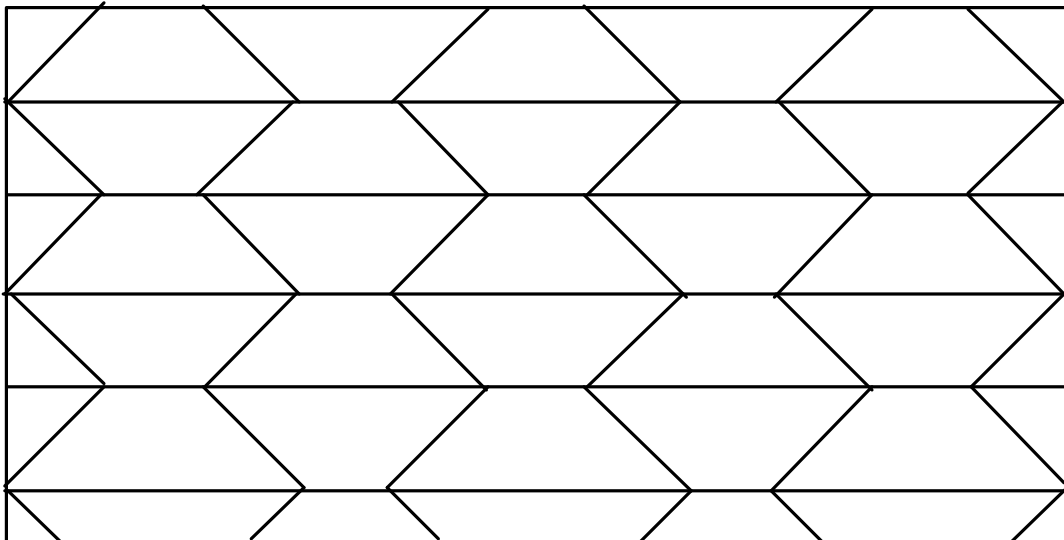
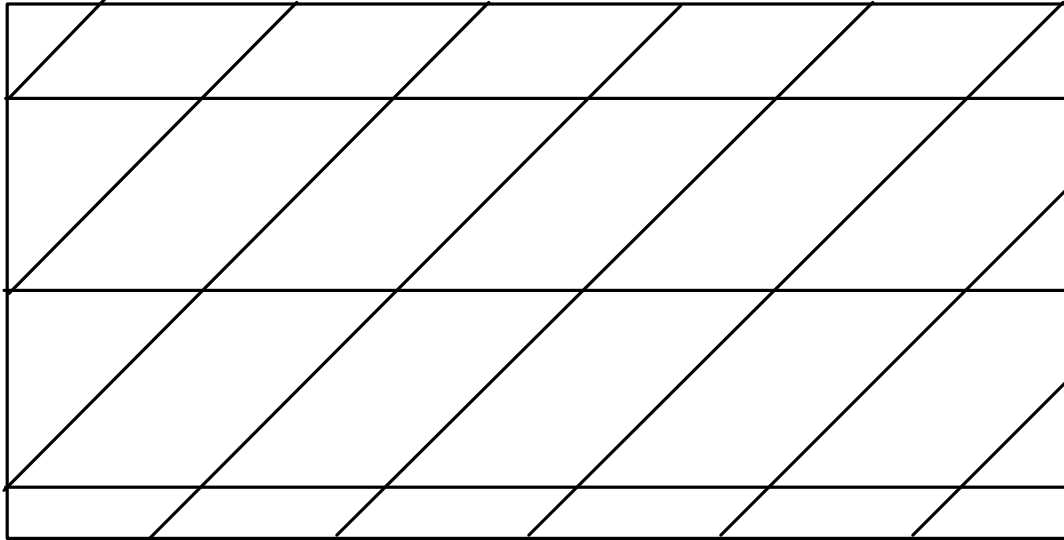
Page 1





Tessellations of Quadrilaterals

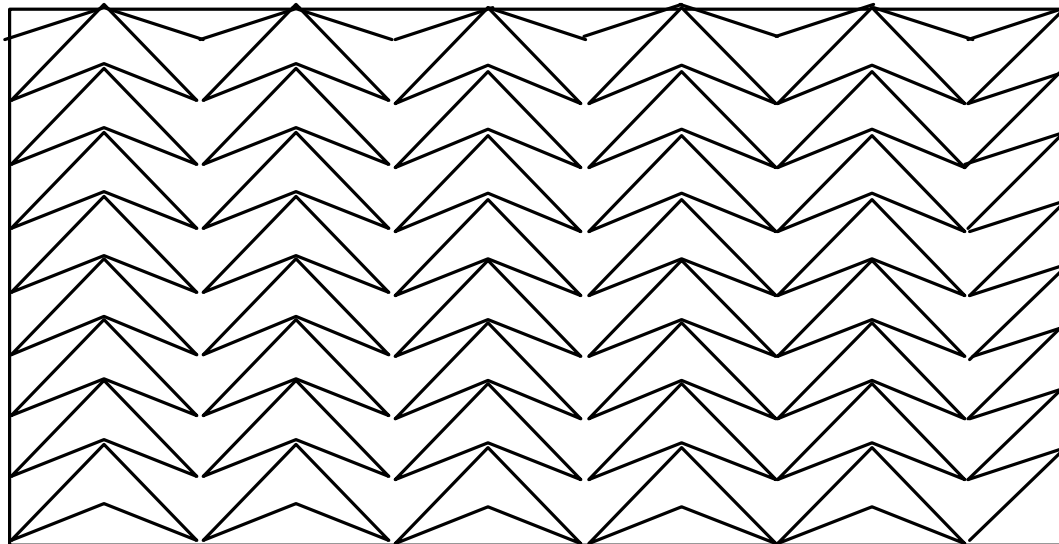
Page 2





Tessellation of Quadrilaterals

Page 3





Activity: Tessellations by Translation

Format: Individual

Objective: Participants will create their own tessellations using translations.

Related SOL: 8.8

Materials: One large square piece of paper, straightedge, scissors, Tessellations by Translation Activity Sheet, Square Activity Sheet

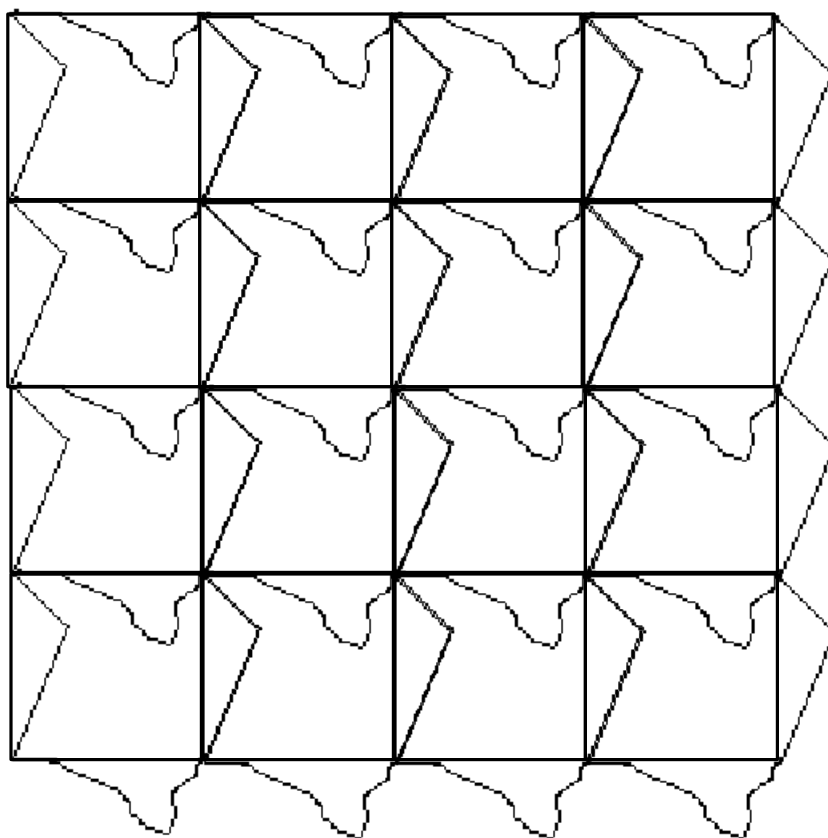
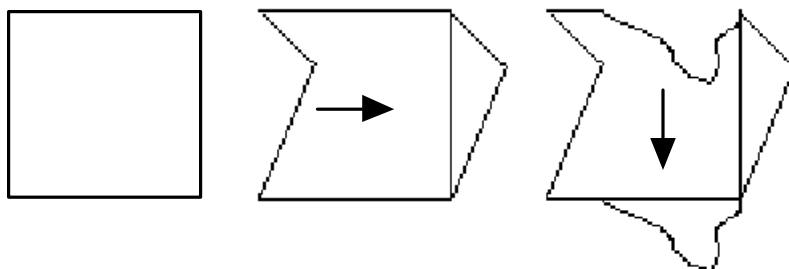
Time required: Approximately 40 minutes

Directions:

- 1) Sketch a figure extending into the square from one side on the Square Activity Sheet. Cut out that figure from the one side of the square and slide it across to the other side. Trace it as shown on the Tessellations by Translation Activity Sheet.
- 2) Sketch a figure extending into the square from the top on the Square Activity Sheet. Cut out that figure from the top of the square and slide it down to the bottom. Trace it as shown on Tessellations by Translation Activity Sheet.
- 3) Cut out the other side and the bottom of the square along the figures you traced. Now you have your pattern to tessellate.
- 4) Trace the pattern repeatedly, translating it as shown on Tessellations by Translation Activity Sheet to form a tessellation.

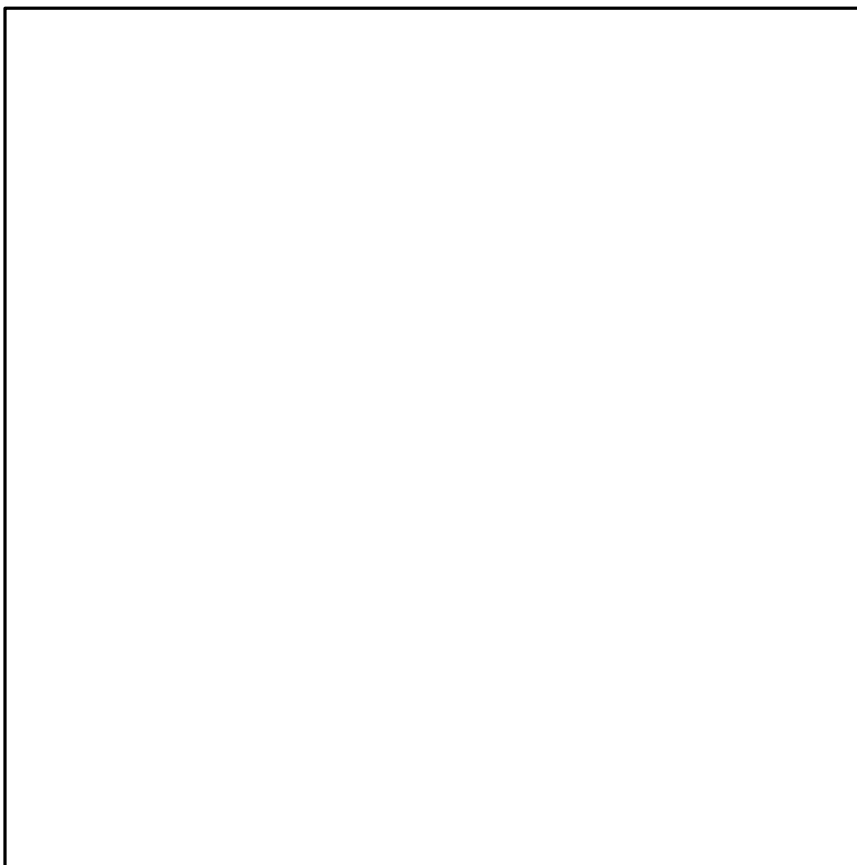


Tessellations by Translation





Square





Activity: Tessellations by Rotation

Format: Individual

Objective: Participants will create their own tessellations using rotations.

Related SOL: 8.8

Materials: One large square piece of paper, scissors, Tessellations by Rotation Activity Sheet, Square Activity Sheet (from previous activity)

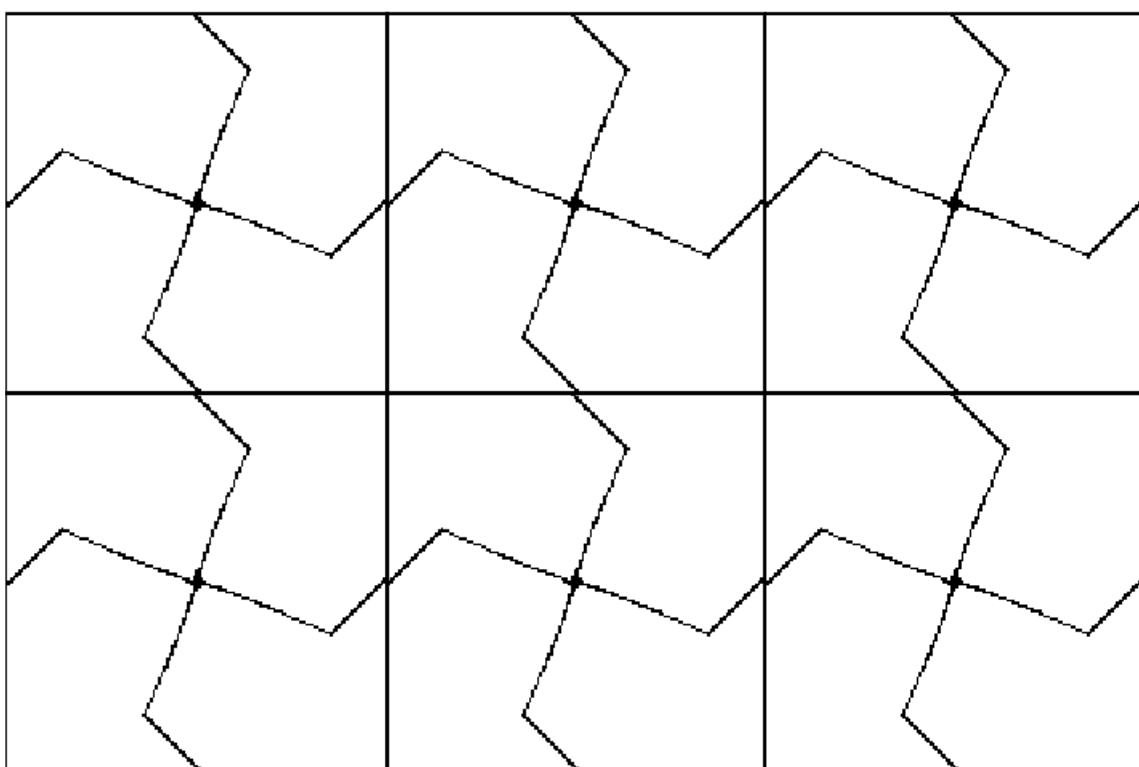
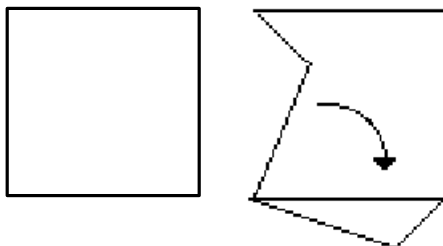
Time required: Approximately 40 minutes

Directions:

- 1) Sketch a figure extending into the square from one side on the Square Activity Sheet. Cut out that figure from the one side of the square and rotate it across to the other side. Trace it as shown on Tessellations by Rotation Activity Sheet.
- 2) Cut out the figure you traced and the other sides of the square. Now you have your pattern to tessellate.
- 3) Trace the pattern repeatedly, translating it as shown on Tessellations by Rotation Activity Sheet to form a tessellation.



Tessellations by Rotation





Middle School Geometry Session 4

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Circles	Circumference vs. Diameter	106	6.12		Can of tennis balls and piece of string
	Cake Problem	107	6.12	Cake Problem	
	Problem Solving with Circles	112	6.12, 7.8, 8.10	Exploring Circumference and Perimeter Problem Solving	Calculators
Pythagorean Theorem	Geoboard Exploration of the Pythagorean Relationship	118	8.10	Geoboard Exploration of the Pythagorean Relationship	11 pin geoboard or dot paper
	Egyptian Rope Stretching	120	8.10		Rope that has 13 knots tied in equal intervals
Solid Geometry	Polyhedron Sort	122	6.17		1 set of geometric solids per group of 4-6
	What's My Shape? Ask About Me	123	6.17		
	What's My Shape? Touch Me	124	6.17		
	Creating Nets	125	6.17	Creating Nets	Cardboard cereal boxes, canisters, milk cartons, etc.,
	Building Polyhedra	127	6.17		Scissors and tape, or a commercial 3-D building kit
	6 Flexagon	130	7.8, 8.7	Flexagon Fun	Strips of sturdy paper about 2" wide and model
	Cover It Up	132	7.8, 8.7	Cover It Up	Pricing and coverage info from a paint store, yard/meter stick
	Architect's Square	135	6.17, 7.8, 8.9	Memoranda 1, 2, 3	Unit cubes (60/group), Isometric dot paper, supplies for making brochures



Topic: Circles

Description: Participants will explore relationships between the elements of a circle, including applied problems such as the relationship of circumference of a tennis ball relative to its diameter. Then they solve a problem that relates radius to area of a circle. They will use a dynamic geometry software program and a graphing calculator to approximate pi. They will use paper folding to investigate tangents to circles and inscribed angles.

Related SOL: 6.12, 7.8, 8.10



Activity: **Circumference vs. Diameter**

Format: Whole class demonstration with small group and whole class discussion

Objectives: Participants will measure the circumference of a tennis ball and compare it to the diameter of the ball.

Related SOL: 6.12

Materials: A can of tennis balls and a piece of string

Time Required: 15 - 20 minutes

Directions:

- 1) Bring a tennis ball can to class. Ask participants which is greater, the height of the can or the circumference of the can? Have each participant write down his/her answer supported with reasons. Encourage small group discussion before having a whole class discussion.
- 2) Discuss participant conclusions with the whole class.
- 3) Wrap a string around a tennis ball and compare this to the height of the can. Discuss results.



Activity: **Cake Problem**

Format: Small group

Objectives: Participants will solve a real world problem using knowledge of area of circles.

Related SOL: 6.12

Materials: Cake Problem Activity Sheet

Time Required: 15 - 20 minutes

- Directions:**
- 1) Present the cake problem to participants. Make sure everyone understands the problem. Put a 10-inch diameter circle on the overhead projector to model the cake and ask a volunteer to make an estimate of the placement of the cut that solves the problem.
 - 2) Give out the handout and have participants work in small groups to solve it. Have participants write up their solution.
 - 3) Ask for volunteers to share solutions. Discuss variations on solutions.
 - 4) Return to the transparency of the cake and draw the correct solution. How close did the estimate come?



Cake Problem

You have a cake that is 10 inches in diameter. You expect 12 people to share it, so you cut it into 12 equal pieces. (See Figure A).

Before you get a chance to serve the cake, 12 more people arrive! So you decide to cut a concentric circle in the cake so that you will have 24 pieces. (See Figure B).

How far from the center of the cake should the circle cut be made so that all 24 people get the same amount of cake?

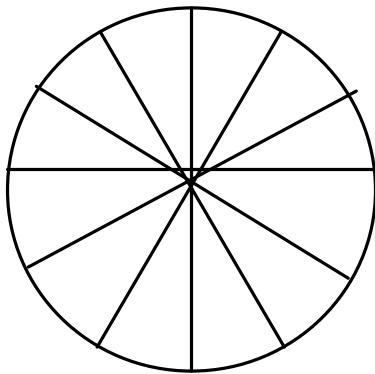


Figure A

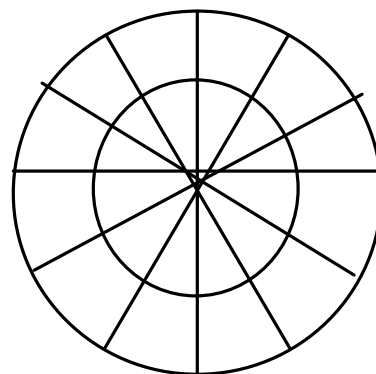


Figure B



Activity: **Problem Solving With Circles**

Format: Small group

Objectives: The participant will solve problems involving circumference of a circle and the volume of a cylinder.

Related SOL: 6.12, 7.8, 8.10

Materials: Calculators, Exploring Circumference and Perimeter, and Problem Solving Activity Sheets

Time Required: 45 minutes

Directions:

- 1) Refer to Exploring Circumference and Perimeter Activity Sheet and discuss the expectations with participants. Have them work in small groups to complete the problems. Discuss their solutions. (See the Teacher's Guide on the next page.)
- 2) Refer to the Problem Solving Activity Sheet and discuss it with participants. Let them work in groups to solve the problems. Then discuss the solutions with the whole class.



Exploring Circumference and Perimeter Teacher Guide

Using the Activity:

In this activity, participants can use the calculator to find the lengths of three different ramps in-line skaters might use. Participants will use the pi, square, and square-root keys on the calculator. The teacher may want to review the formula for finding the circumference (perimeter) of a circle. Discuss how to find the lengths of various arcs of the circle. To find the length of the third ramp, participants will need to use the Pythagorean Theorem.

Another important extension to this activity is finding the steepness of the various ramps. Participants can use this data to determine the level of difficulty of each ramp.

Answers:

First ramp: 51.41592654 ft

The length of the arc AB is one quarter the circumference of the circle.

Second ramp: 40.943951 ft

The length of the arc is one-sixth the circumference of the circle.

Third ramp: 41.540659 ft

The length of the ramp x is found by using the Pythagorean theorem.

Discussion Questions:

- 1) What makes one ramp better than another?
- 2.) Which ramp is safest? Why?
- 3.) Which construction is more challenging? Why?

Thinking Cap:

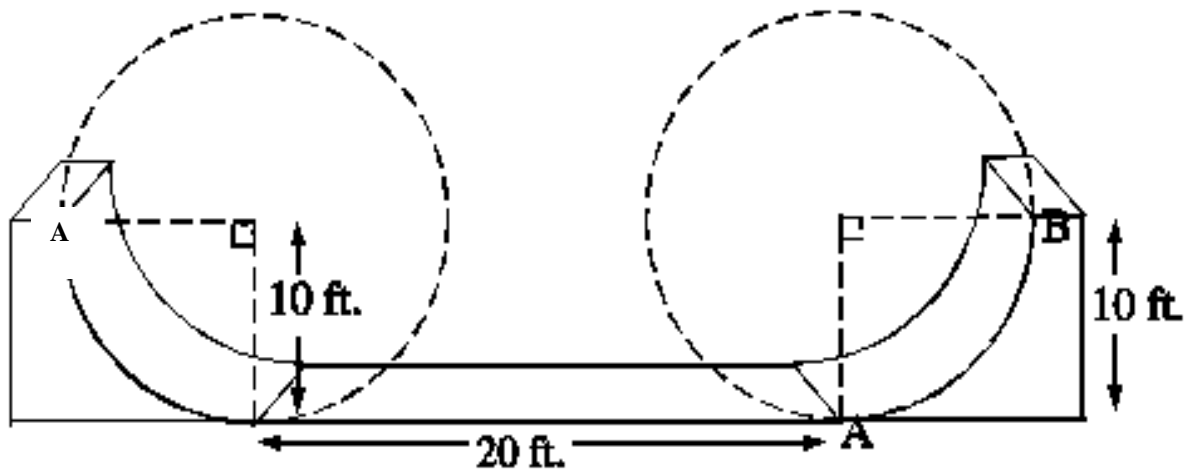
$$D = 2L + (2)(1/2)(\pi)d = 2L + (\pi)d$$

(Adapted from lesson by Ann Mele, Assistant Principal, Offsite Educational Services; NY Public Schools; NY, NY found at <http://pegasus.cc.ucf.edu/~ucfcasio/activities/line.htm>.)



Exploring Circumference and Perimeter

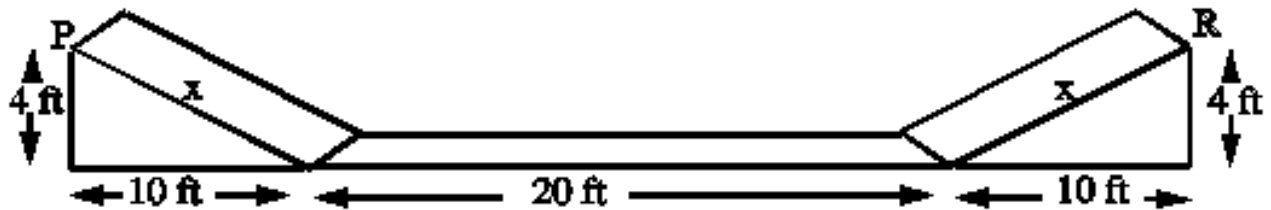
In-line skating has become a popular city sport. The parks department is thinking of constructing ramps in some of the local playgrounds. A "half-pipe" ramp is formed by two-quarter circle ramps each 10 feet high with a flat space of 20 feet between the centers of the circles from which the two-quarter circle ramps are formed.



- 1) Find the distance a skater travels from the top of one ramp to the top of the other. (Hint: What is the length of AB ?)
- 2) Another launch ramp is formed by 2 arcs each with a central angle of 60 degrees and a radius of 10 ft. Find the length from the top of one ramp to the top of the other. (Hint: What fractional part of the circle is each arc?)



3. A third ramp is a straight ramp 4 ft high and 10 ft long with a flat space of 20 ft. Find the length of the ramps from point P to point R. (Hint: Use the Pythagorean Theorem)



Thinking Cap

A school track is formed by 2 straight segments joined by 2 half circles. Each segment is L long and each half circle diameter is D in length. Write a formula for finding the distance, D , around the track.



Problem Solving

SODA STRAWS

How many straws full of pineapple juice can be taken from a 46 fl. oz. can of juice that is filled to the top?

diameter of the can _____

height of the can _____

diameter of straw _____

length of straw _____

Explain how you determined your answers.

DUCT TAPE

How many rolls of duct tape would it take to create a one-mile strip of tape? (You may not unroll the duct tape or read its length from the packaging). You may experiment with one 6-inch piece of tape.

1) Imagine tape as concentric circles...1st method

2) Find volume of tape...2nd method

(Adapted from lesson by Bob Garvey, Louisville Collegiate School, Louisville, KY)



Topic: **Pythagorean Theorem**

Description: Participants will engage in experiences that allow them to verify the Pythagorean Theorem and its converse. They are guided through several variations of proofs of the Theorem.

Related SOL: 8.10



Activity: **Geoboard Exploration of the Pythagorean Relationship**

Format: Whole class and small groups

Objectives: The participant will explore the Pythagorean relationship by constructing right triangles on a geoboard and verifying the Pythagorean Theorem.

Related SOL: 8.10

Materials: 11-pin geoboards or dot paper, Geoboard Exploration of Right Triangles Activity Sheet

Time Required: 30 minutes

- Directions:**
- 1) On a transparent geoboard on the overhead projector, construct a right triangle in which one leg is horizontal and the other is vertical. Ask a participant to construct a square on each leg and then on the hypotenuse of the triangle. Ask participants to find the area of each square. It may be difficult for some participants to recognize a way to find the area of the square on the hypotenuse, so you may need to assist them.
 - 2) Refer to the Geoboard Exploration of Right Triangles Activity Sheet and have them fill in the data from the example that was done by the whole class. Then have small groups work to find several other examples and record them in the chart.
 - 3) Using the Activity Sheet, debrief the examples with the whole class:
 - What patterns do you see? (If participants have not seen it before, you may tell them that this is the Pythagorean relationship that will be stated later as a theorem.)
 - Can you state the relationship in words? In symbols?
 - Do you think this is always true?
 - If you label the sides of the triangle, can you write a statement of what you think is true?
 - Does this procedure provide a proof that the relationship is always true?



Geoboard Exploration of Right Triangles

Make a right triangle on a large geoboard or dot paper. Construct a square on each side of the triangle. Label the shortest side a , the middle side b and the longest side (hypotenuse) c .

Complete the Table

Length of side a	Length of side b	Length of side c	Area of square on side a	Area of square on side b	Area of square on side c	$a^2 + b^2$



Activity: Egyptian Rope Stretching

Format: Large Group

Objectives: The participant will study an artifact of ancient Egyptian culture and will see how it was used as an application of the Pythagorean Theorem.

Related SOL: 8.10

Materials: A rope that has 13 knots tied at equal intervals

Time Required: 30-40 minutes

- Directions:**
- 1) Show participants the rope that has 13 knots tied at equal intervals. Tell them that a picture of a rope like this was found on inscriptions in tombs of ancient Egyptian kings. Ask participants to work in groups to figure out what the purpose of the rope might have been.
 - 2) Ask participants to suggest ideas for the way the Egyptians used the rope. If participants come up with the idea that it was used to make a template for determining right angles, let them demonstrate. If they do not, you should have two participants help you demonstrate. Have a participant hold knot #1 and knot #13 together. Have another hold a different knot. You should hold knot # 4. All three of you should stretch the rope and have the class observe the resulting shape. Make sure they understand that the only shape that can be formed when you pull is a right triangle.
 - 3) Have participants conjecture about how the Egyptians might have used this rope. (To build right angled corners on pyramids? To redraw the boundaries of the fields after the spring flooding of the Nile?)
 - 4) Ask participants what other numbers of knots in a rope might be used to serve the same purpose of forming a right triangle. For example:
 - Could you obtain the right triangle result with a rope that has 20 equally spaced knots? Try it.
 - Could you do it with a rope with 31 knots (30 spaces)? Why or why not?



Topic: **Solid Geometry**

Description: Participants will explore three-dimensional shapes by sorting and classifying them, determining what they are by touch, building them, and taking them apart.

Related SOL: 6.17, 7.8, 8.7, 8.9



Activity: **Polyhedron Sort**

Format: Small Group/Large Group

Objectives: Participants sort polyhedra in three different ways and compare their sort to what they predict their participants would do. They then relate the sorts to the van Hiele levels of geometric understanding of polyhedra.

Related SOL: 6.17

Materials: One set of geometric solids per group of 4-6

Time Required: Approximately 10 minutes

- Directions:**
- 1) Divide the participants into small groups. Pass out the sets of geometric solids, one set per 4-6 participants. Have them sort the solids into groups that belong together, sketching the pieces they put together and the criteria they used to sort. Have them sort two or three times, recording each sort.
 - 2) Ask the participants for some of their ways of sorting. Have them compare their ways with those of other groups.



Activity: **What's My Shape? Ask Me About It.**

Format: Large Group

Objectives: Participants will use logical reasoning to determine which polyhedron is in the box after asking questions and receiving information about it.

Related SOL: 6.17

Materials: One set of geometric solids, box or bag

Time Required: Approximately 10 minutes

Directions:

- 1) The teacher says to the participants, "This box (or bag) contains a polyhedron." Shake it so the participants can hear. "I'd like you to ask me some questions with yes or no answers to figure out what is in the box."
- 2) Questions and answers continue until the participants can figure out what shape is in the box.



Activity: **What's My Shape? Touch Me.**

Format: Large Group

Objectives: Participants will determine which polyhedron is in the bag by touch alone.

Related SOL: 6.17

Materials: One set of geometric solids, a paper bag

Time Required: Approximately 10 minutes

Directions:

- 1) The teacher says to the participants, "This bag contains a polyhedron." Shake it so the participants can hear. "One of you at a time may put your hand into the bag and touch the solid. Try to figure out what is in the bag."
- 2) The participants take turns touching the solid in the bag without looking and try to figure out what the solid is.



Activity: **Creating Nets**

Format: Small Group /Large Group

Objectives: Participants will determine the shapes that form various polyhedra and analyze how they fit together to form the polyhedra.

Related SOL: 6.17

Materials: Cardboard cereal boxes, canisters, milk cartons, etc., emptied and cleaned, scissors, Creating Nets Activity Sheet

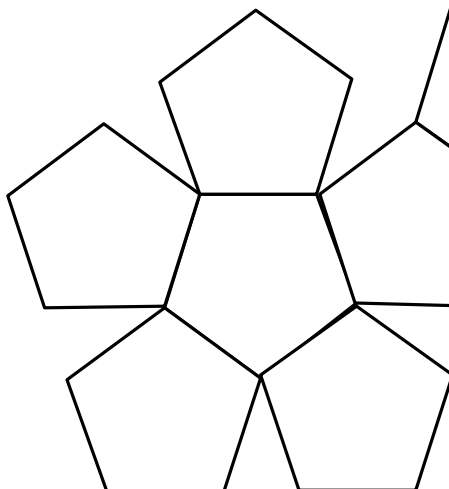
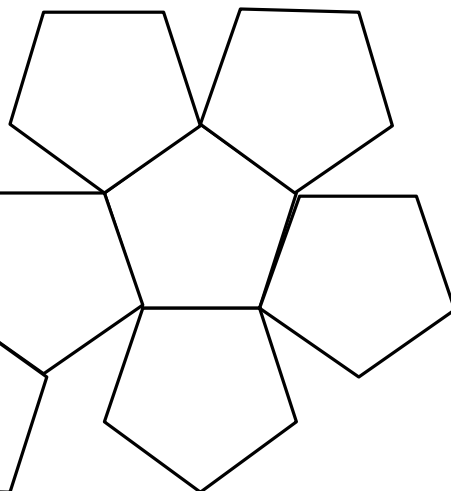
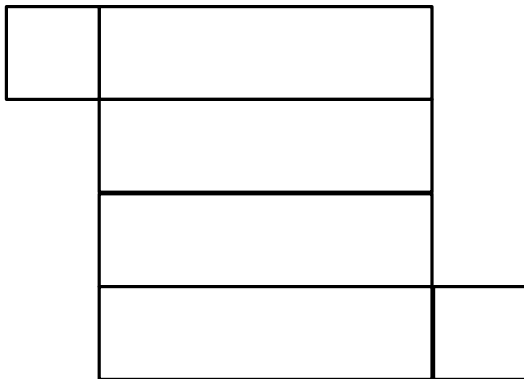
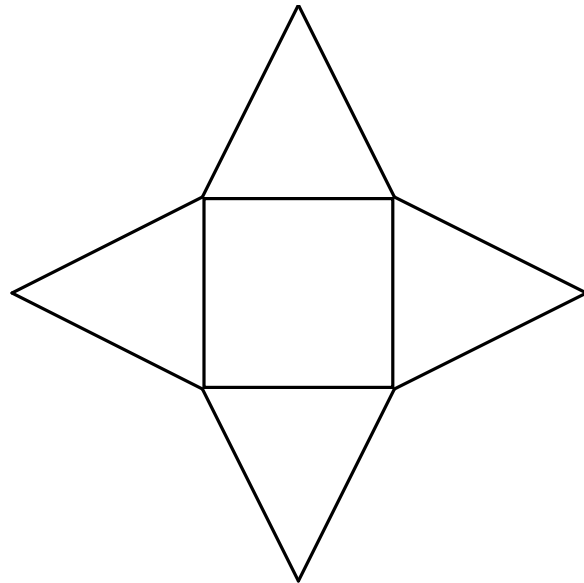
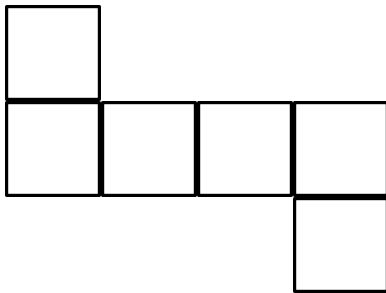
Time Required: Approximately 10 minutes

Directions:

- 1) The teacher or class collects a variety of cardboard containers such as cereal boxes. The teacher (or the participants) carefully cut apart a container along its seams, in such a way that the container can be flattened out, but each piece is connected. This shape is called the **net**. Some nets appear on the following page.
- 2) The class should identify each shape formed.
- 3) The class should see how many different nets they can find for the same container.



Creating Nets





Activity: **Building Polyhedra**

Format: Small Group / Large Group

Objectives: Participants will determine the shapes that form various polyhedra and analyze how they fit together to form the polyhedra.

Related SOL: 6.17

Materials: Scissors, tape or glue, geometric solids, and handout made by tracing the sides of various geometric solids; or a commercial 3-dimensional building kit

Time Required: Approximately 20 minutes

- Directions:**
- 1) Have each participant choose a geometric solid from the set. Have the participant make a handout by tracing each of the faces to form its net. Repeat for 2 or 3 more solids.
 - 2) Pass out scissors, tape or glue, and multiple copies of handout, or a commercial 3-dimensional building kit.
 - 3) Working in small groups, the participants should predict which solid the net will make. The teacher may show several solids, one of which is the one that the net will form.
 - 4) After the predictions are made, the participants should cut out the net and tape it together.
 - 5) The participants should compare their solids to their predictions and to the original solids.



Activity: **6-Flexagon**

Format: Individual

Objectives: Participants will construct a six-flexagon (hexaflexagon) and explore what faces are visible as the structure is flexed.

Related SOL: 7.8, 8.7

Materials: Strips of sturdy paper about 2" wide, model constructed by the teacher, Flexagon Fun Activity Sheet

Time Required: Approximately 30 minutes

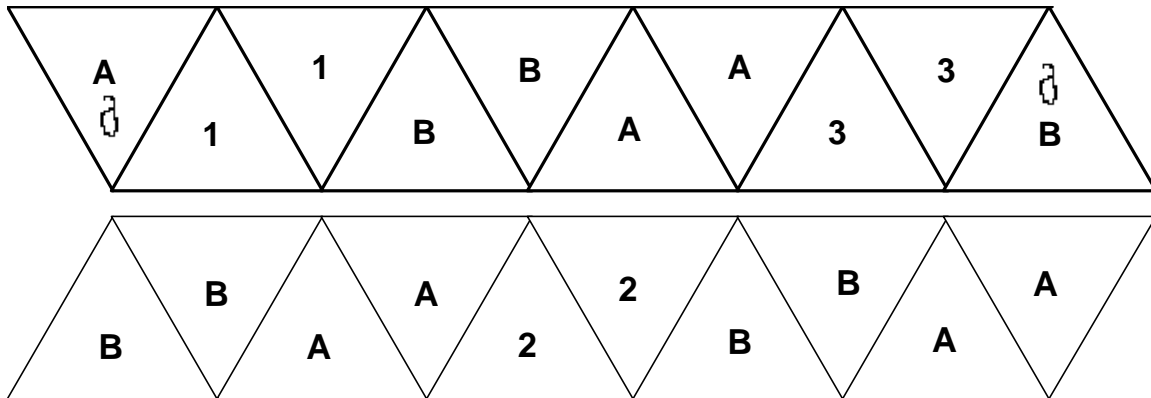
Directions:

- 1) Review the terms mountain fold (the fold is at the top with the paper coming down) and valley fold (the fold is at the bottom with the paper coming up) so that all participants are familiar with the terms.
- 2) Have participants create a flexagon using the pattern and instructions provided on the activity sheet.
- 3) Participants should explore which faces of the flexagon are visible at which times. How can they use this information to make a game of the flexagon?

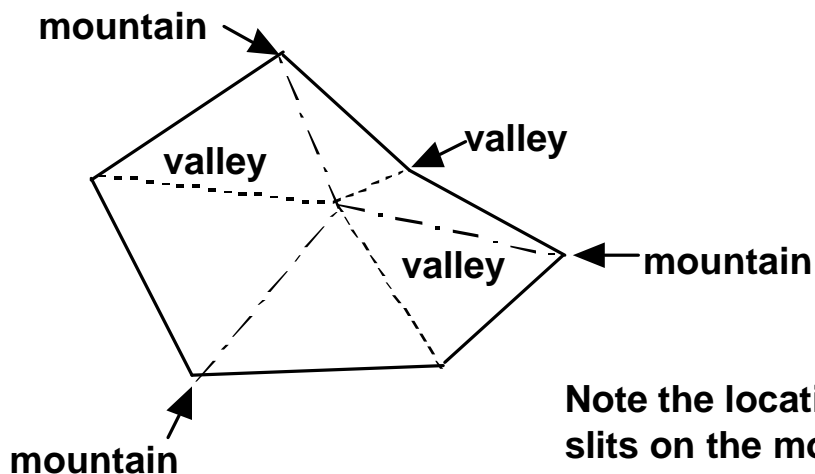


Flexagon Fun

- 1) Using the pattern below, mark your strip of paper as indicated.



- 2) Crease all fold lines (dotted) in both directions so that you will be able to flex your construction easily.
- 3) Fold in order (so that the numbers aren't visible) triangle 1 onto triangle 1, triangle 2 onto triangle 2, triangle 3 onto triangle 3, etc., until you fold a triangle with a teardrop on top of the other.
- 4) Glue or clip the last two (teardrop) triangles.
- 5) Gently flex your model by making mountain and valley folds as shown below. Which faces can you see at any one time? What patterns do you notice between the pattern above and the flexagon you've created?





Activity: **Cover It Up**

Format: Individual or Small Group

Objectives: Participants will calculate the surface area of the classroom. Participants will use this calculation in figuring out materials needed and cost for a classroom-painting job.

Related SOL: 7.8, 8.7

Materials: Cover It Up Activity Sheet, pricing and coverage information from the paint store, measurement tools

Time Required: Approximately 2-3 class periods

- Directions:**
- 1) In the first class period, participants will calculate the surface area of the classroom. The problem can be made more or less complex by including windows and doors (or not) and by painting the ceiling and/or floor or not. The activity sheet assumes that walls and ceiling are painted and that windows and doors are subtracted from the wall surface area. It will need to be modified if other assumptions are made.
 - 2) Once the participants know how much surface is to be covered, they should use the information from paint stores about coverage and cost to figure how much paint is needed.
 - 3) Finally, participants should write a formal proposal for the job explaining what their cost estimate is and how they arrived at this figure. You may wish to require them to include a diagram.



Cover It Up

As an independent painting contractor, you have been asked to provide an estimate for the cost of painting your classroom. In order to prepare your estimate, you need to sketch the classroom, calculate the surface area, and figure the cost of paint. We suggest the following process for calculating your estimate.

- 1) Sketch a bird's eye view of the classroom in the space below. Be sure to mark which walls have windows and doors.

- 2) Measure each wall, window, and door and mark the measurements on your diagram.

- 3) List each wall, window, and door below and calculate its area.

Object	Length	Width	Area

Extend this table as needed.

- 4) Find the area of the ceiling. It must be painted as well.
- 5) Find the total area of the walls and ceiling by adding each area together.
- 6) Find the total area of windows and doors. Subtract this area from the total you found in step 5 to find the area you need to paint.



GEOMETRY

- 7) A paint can includes information about cost and coverage. You will want to put two coats of paint on the walls and ceiling. This means you will cover the area you found in step 6 twice.
- 8) Using the information about coverage and cost, find the number of cans of paint you need and the cost for that number of cans.
- 9) Finally, write a letter to your teacher submitting your bid. Your letter should be carefully written (neatly, with correct grammar and spelling) and should include the following information:

How did you calculate the area to be painted?

How many cans of paint are required?

How much will the paint cost?



Activity: Architect's Square

Format: Small Group

Objectives: Participants will create all possible arrangements of four unit cubes. Participants will draw these structures on isometric paper. Participants will make accurate calculations about the surface area of these figures.

Related SOL: 6.17, 7.8, 8.9

Materials: Unit cubes (approximately 60 per group); isometric dot paper; Memoranda to Architects 1, 2, and 3; Architect's Square Activity Sheet, supplies for making brochures (optional)

Time Required: Approximately 5 class sessions

- Directions:**
- 1) Have the participants work in groups of 2-4. Instruct them to build all possible arrangements of four unit cubes. These will be houses, and the first memorandum sets the stage for the activity. Participants should record their figures on isometric dot paper. (2 days)
 - 2) Once participants have found all 15 possible arrangements, they should use the information in Memorandum #2 to calculate costs for each house. (1-2 days)
 - 3) Using their information, participants should next plan a brochure advertising their houses. They should think about the features of each house for an elderly couple, a single person, and a family with small children. Memorandum #3 contains this information.



Memorandum #1

TO: All Architects

RE: Housing Designs

We are beginning work on a new subdivision of modular homes. Each home will include four units. All units are shaped as cubes and adjacent units must touch over a full face. It is not acceptable to have faces overlap partially, nor is it acceptable for cubes to touch at edges only.

Your task is to figure out how many different houses we can design. A design is considered different from another if they cannot be superimposed without reflection. Use the cubes provided to you to design and record (on dot paper) all possible home designs.



Memorandum #2

TO: All Architects

RE: Housing Costs

Now that you have drawn all possible houses, you must figure the cost of each house. There are three factors that influence the cost of our houses. Each square unit of land has a fixed cost, each square unit of wall has a fixed cost, and each square unit of roof has a fixed cost. These costs are as follows:

Land	\$10,000 per square unit
Wall	\$ 5,000 per square unit
Roof	\$ 7,500 per square unit

Using your drawings, models you build, and these figures, complete a chart like the one below to price each home.

Design #	Units of Land	Land Cost	Units of Wall	Walls Cost	Units of Roof	Roof Cost	Total Cost



Memorandum #3

TO: All Architects

RE: Advertising

Now that your houses are designed and costs calculated, we need your help with a marketing plan. There are three groups we would like to target — elderly couples, families with small children, and single people. Look at your design and consider such factors as stairs to climb, space for people to spread out, and cost. Which home design would be best for each of these three groups?

Plan a brochure that includes designs for at least six different homes, including the three you selected above. For each home, include a sketch of the home on dot paper, the calculations for the cost of the home, and three to five selling points for that design. Your brochure should be colorful and easy to read.



Middle School Geometry Session 5

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Perimeter And Area, Part I	Dominoes and Triominoes	141	6.14, 6.15, 8.8	One-Inch Grid Paper	Tiles, scissors
	Tetrominoes	144	6.14, 6.15, 8.8	One-Inch Grid Paper, Sorting Mat	Tiles, scissors, paste (optional)
	Pentominoes	147	6.14, 6.15, 8.8, 8.9	Pentomino Alphabet, Solutions Sheet	Tiles, scissors, and envelopes or plastic baggies
	Areas with Pentominoes	151	6.14, 6.15, 7.7, 8.8	Pentomino Puzzles, 3 x 5 Solutions, 4 x 5 Solutions, Patterns for Pentominoes 1 and 2	Scissors and envelopes or plastic baggies; or sets of Pentominoes
	Hexominoes	158	6.14, 8.7, 8.8	Hexominoes 1, 2, and 3	Tiles, scissors, grid paper, and envelopes
	Perimeters with Hexominoes	162	6.11, 6.14, 7.7	Perimeters of Hexominoes	Hexominoes from previous activity
Perimeter And Area, Part II	The Perimeter is 24 Inches. What's the Area?	165	6.11, 6.14, 7.7, 8.8	One-Inch Grid Paper, One-Half-Inch Grid Paper	24-inch paper strip (collar), 38 one-inch cubes, grid paper
	The Area is 24 Square Inches. What's the Perimeter?	168	6.11, 6.14, 7.7, 8.8		24 one-inch squares and grid paper
	Change the Area	169	6.11, 6.14, 7.7	Geoboard Dot Paper	Square geoboard, overhead geoboard, and rubber bands



Topic: Perimeter and Area, Part I

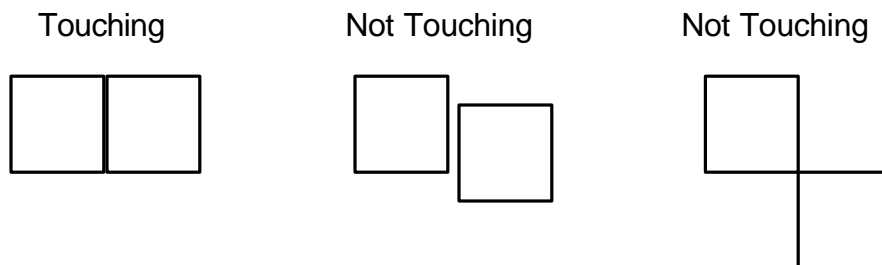
Description: Participants will explore the concept of area through the use of "-ominoes" formed by arranging sets of squares. Putting two squares together, with full edges touching, forms the domino, the most familiar of the "-ominoes." Using the "-ominoes" rule (see below), there is only one figure that you can make with two squares. However, by adding squares, other "-ominoes" can be produced. Working with "-ominoes," the participants will have the opportunity to:

- use spatial visualization to build figures;
- develop strategies for finding new figures;
- test to find figures that are flips and turns of other figures;
- reinforce their understanding of congruence by flipping and turning figures in order to compare them;
- use "-ominoes" to fill given rectangular areas; and
- sort "-ominoes" according to their perimeter and look for patterns in their collected data.

A unit on "-ominoes" can be developed through the use of materials such as color tiles, paper squares, the orange block in the pattern block set, interlocking cubes, or grid paper. Most "-ominoes" activities are best done where children work in groups of two to four, each group with its own set of materials. In that format, groups can explore the various problems independently. All activities should be conducted in an easy-going manner that encourages risks, good thinking, attentiveness, and discussion of ideas.

"-Ominoes" Rule:

At least one full side of each tile must touch one full side of another tile.



Related SOL: 6.11, 6.14, 6.15, 7.7, 8.7, 8.8



Activity: Dominoes and Triominoes

Format: Large Group/Small Group

Objectives: Participants will use spatial visualization to build "-ominoes" with two and three tiles. They will test to find figures that are the result of reflections and rotations.

Related SOL: 6.14, 6.15, 8.8

Materials: Tiles, scissors, grid paper preferably with the same size squares as the tiles, One-Inch Grid Paper Activity Sheet

Time Required: Approximately 20 minutes

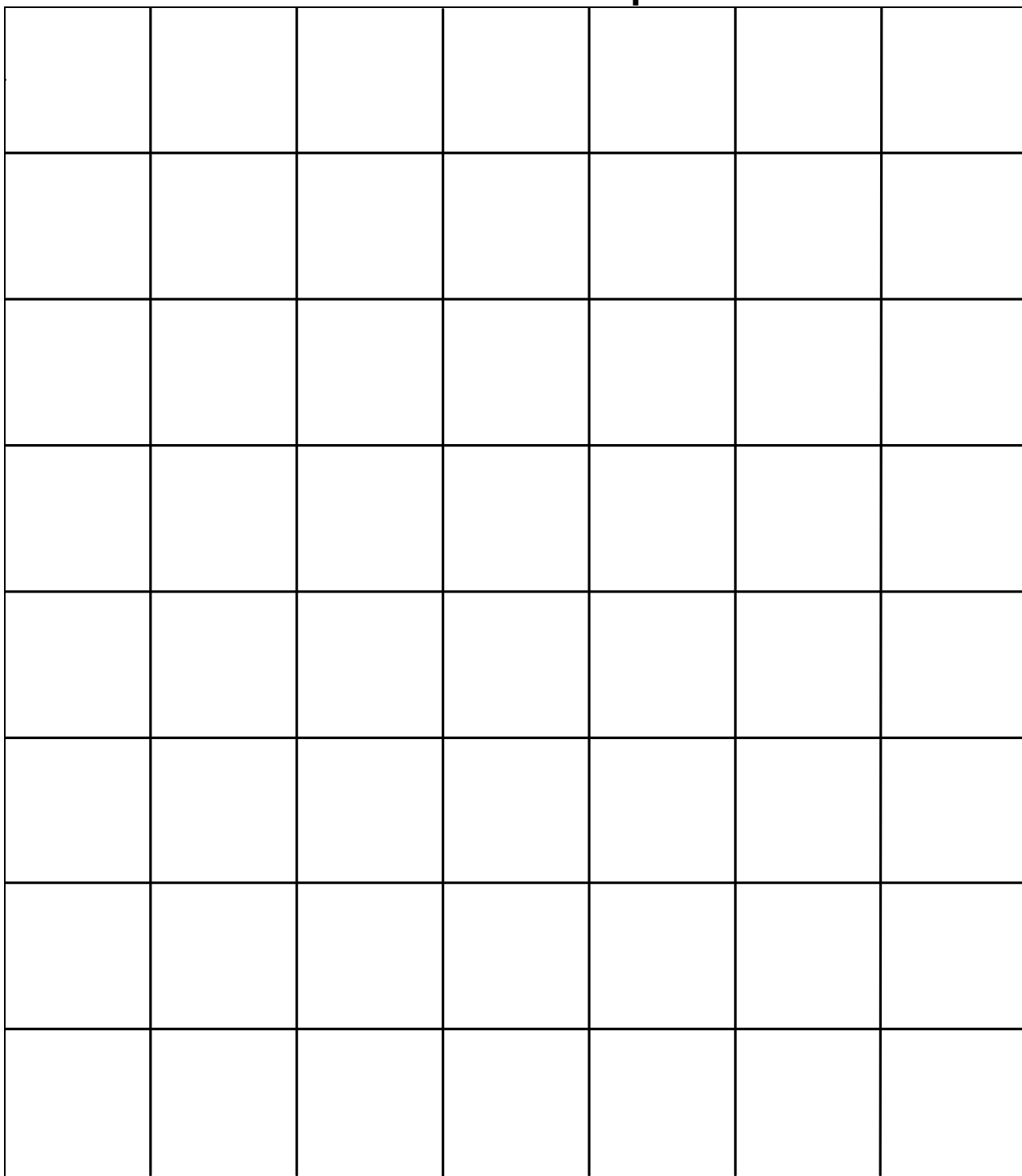
- Directions:**
- 1) Show the participants a **domino** and ask them to describe everything they can about the domino. Putting two squares together, with full edges touching, forms this figure. There is only one figure that you can make with two squares. Establish that it's made up of two tiles and that each tile must have one side in common with the other tile.
 - 2) Ask the participants if they can find another way to arrange the two tiles so that every tile touches at least one complete side of another tile. They may describe the 90-degree rotation as different. As they discuss their strategies for determining whether or not two figures are the same, you may want to informally introduce the terms *flip* (reflect) and *turn* (rotate) used in transformational geometry.
 - 3) Ask the participants to arrange three tiles in as many ways as possible and to tell something about the figure. Establish that it is made up of three color tiles and that every tile shares at least one complete side with another tile. Tell them that this is called a **triomino**, noting that the "tri" stands for three as in triangle.



- 4) Ask the participants to find another way to arrange the three tiles so that every tile touches at least one complete side of another tile. Have volunteers who think they have found different figures display them.
- 5) Model how to record the first figure on grid paper and cut it out. Then have the participants record and cut out their figures. Collect the cutout figures, making two piles, one for each of the two possible arrangements. Show how, by flipping (reflecting) or turning (rotating) the pieces in each pile, you can fit them over each other exactly to make a neat stack.



One-Inch Grid Paper





Activity: Tetrominoes

Format: Large Group/Small Group

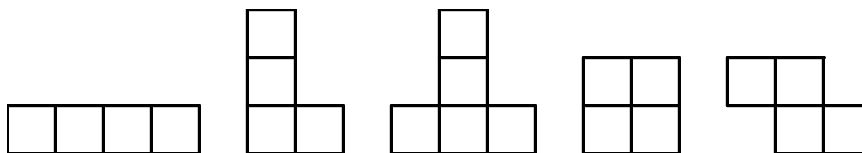
Objectives: Participants will use spatial visualization to build "tetrominoes" with four tiles, testing for figures that are flips and/or turns of other figures.

Related SOL: 6.14, 6.15, 8.8

Materials: Tiles, scissors, paste (optional), One-Inch Grid Paper Activity Sheet from previous activity, Sorting Tetrominoes Activity Sheet

Time Required: Approximately 15 minutes

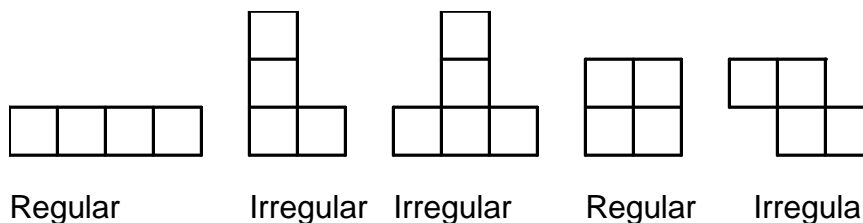
- Directions:**
- 1) Ask the participants to work in pairs. Distribute at least 30 color tiles per pair, scissors, Grid Paper and Sorting Tetrominoes Activity Sheets. Ask each participant to make a figure with four color tiles that follows the rule each square tile has one side in common with another square tile. These figures are called **tetrominoes**.
 - 2) Ask the partners to compare their figures. If both made the same figure, have them copy it only once onto the grid paper and cut it out. If they made two different figures, have them copy and cut out both figures. Ask the participants how many different figures they think they can make with four color tiles.
 - 3) Ask the participants to make more figures with four color tiles. Each time they should compare the figures and record and cut out each different figure. The partners should continue until neither can think of any new figures to make and then count all their different figures, recording that number.



- 4) To promote thinking and sharing, use prompts such as these to promote class discussion:
 - How did you and your partner decide whether or not your figures were different? (Be sure to discuss flips [reflections], turns [rotations], and congruency here.)
 - Did you use any of your old figures to find new ones? If so, how did you do this?
 - How did you know that you found all of the different figures?
 - How many different figures are there? (5)



- 5) Invite participants to determine which of the tetrominoes are regular and which are irregular. (**Regular tetrominoes** are those that are squares or rectangles, while irregulars are not.)



This is another way to reinforce the attributes of squares and rectangles. You might contrast this use of the "regular" with its more general meaning. A **regular polygon** is a polygon that has all congruent sides and all congruent angles. So a rectangle is a regular tetromino, but not a regular polygon. A square is both a regular tetromino and a regular polygon. With teachers, be sure to point out that the definition of regular tetromino is actually redundant because since a square is a type of rectangle, it would be enough to say that a regular tetromino is a rectangle. (Ask at what level this distinction should be discussed with students.) Invite participants to come to the front of the class and place their tetrominoes into one of the categories on the Sorting Tetrominoes Activity Sheet and give the reason for their decision.

- 6) Have participants trace or glue their tetrominoes into the appropriate columns on the Sorting Mat Activity Sheet.



Sort your figures into regular tetrominoes and irregular tetrominoes. If an arrangement of squares does not form a rectangle or a square, then it is irregular. Trace or paste the tetrominoes into the proper column below.

Irregular Tetrominoes

[illegible]



Activity: Pentominoes

Format: Large Group/Small Group

Objectives: Participants will use spatial visualization to build "pentominoes" with five tiles, testing for figures that are flips and turns of other figures.

Related SOL: 6.14, 6.15, 8.8, 8.9

Materials: Tiles, scissors, envelopes or plastic baggies, One-Inch Grid Paper Activity Sheet (used in Domino and Triominoes Activity), Pentomino Alphabet Activity Sheet, Solutions Sheet

Time Required: Approximately 15 minutes

Directions:

- 1) Distribute the materials. Ask the participants to use five color tiles to find all the possible pentominoes. A **pentomino** is a figure made of five squares, each of which has one side in common with another of the squares.
- 2) Have the participants work in pairs, and use five color tiles of one color to make a pentomino. Record the pentomino on grid paper. Then make a different pentomino. Continue making and recording pentominoes until you cannot make any more that are different.
- 3) Instruct participants to cut out the pentominoes. Compare them by flipping (reflecting) and turning (rotating) them to see if any match exactly. If you find a match, keep only one of them
- 4) Number your pentominoes and put them in an envelope.
- 5) Write the number on the envelope. Exchange envelopes with another pair of participants. Check their pentominoes to see that none of them are the same. Mark any that you think are exactly the same. Return the envelopes. Check your envelope to see if any duplicates were found.
- 6) With partners, discuss ways you could sort your pentominoes.



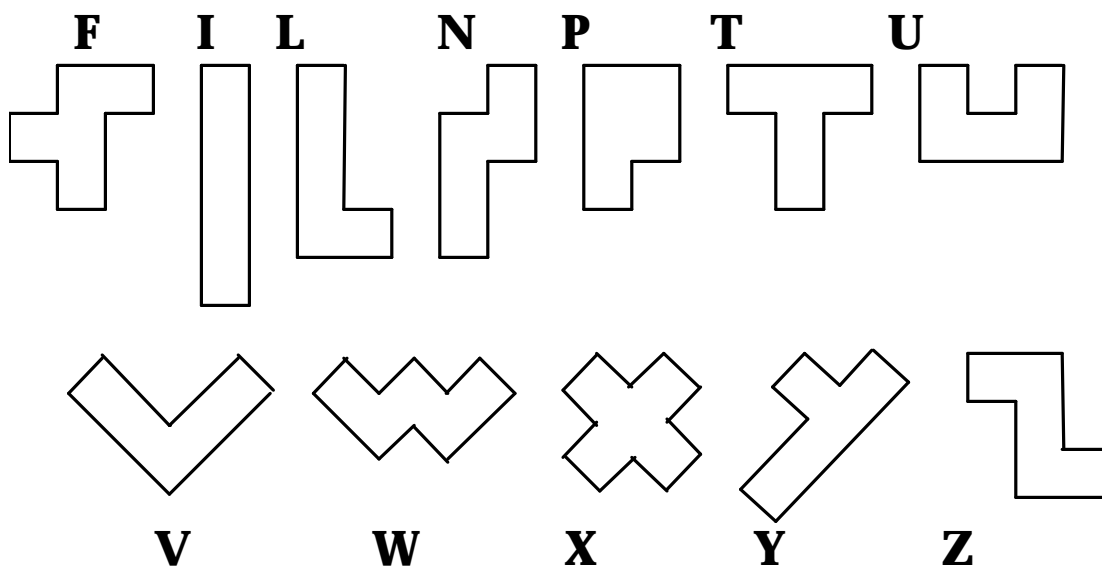
- 7) Ask one or two pairs to post their pentominoes in an organized way. Note the organizations, then discuss each of the posted pentominoes. Point out any duplicates and figures that are not pentominoes. Call on volunteers to supply any missing pentominoes.

Use prompts such as these to promote class discussion:

- Do you think that you have found all possible pentominoes? Explain.
 - In what ways do the arrangements differ from one another?
 - What strategies did you use to make new pentominoes from your old ones?
 - Did you find any patterns while making your pentominoes? Did you use these patterns when sorting them? How?
 - Did you sort the pentominoes according to the letters of the alphabet that they resemble? (Use Pentomino Alphabet Activity Sheet to illustrate.)
 - Did sorting your pentominoes help you find others that were missing? If so, explain.
- 8) Ask the participants to predict which figures can be folded to make topless boxes. Have them put their figures into two groups: those that can be folded into boxes and those that cannot.
- 9) Have the participants fold their figures to test their predictions. Share the Solutions Sheet with participants.



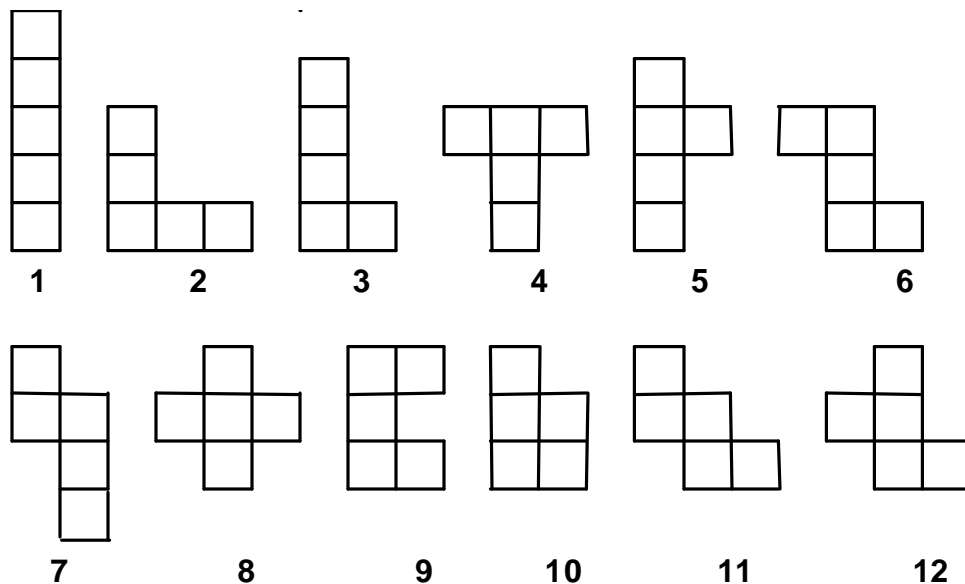
The Pentomino "Alphabet"





Solutions Sheet

Which pentominoes fold up into
an open top box?



Figures 3, 4, 5, 6, 7, 8, 11, and 12
will fold into an open top box.



Activity: Areas with Pentominoes

Format: Large Group/Small Group

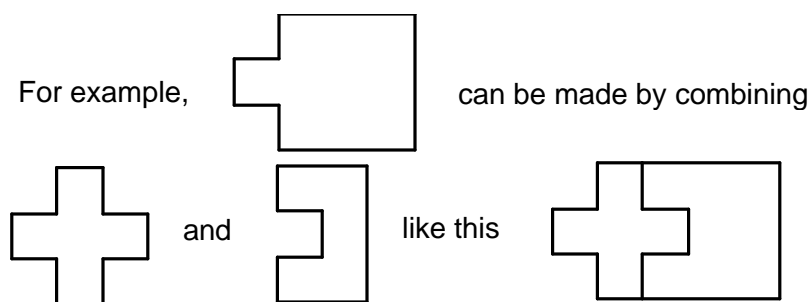
Objectives: Participants will use "-ominoes" to fill given rectangular areas.

Related SOL: 6.14, 6.15, 7.7, 8.8

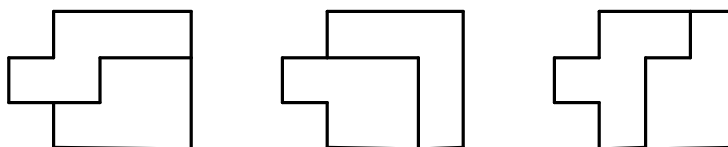
Materials: Scissors, envelopes or plastic baggies or sets of pentominoes made in previous activity, Pentomino Puzzles Activity Sheet, 3 x 5 Solutions Sheet, 4 x 5 Solutions Sheet, Patterns for Pentominoes Sheets 1, 2, and 3

Time Required: Approximately 25 minutes

- Directions:**
- 1) Ask the participants to use the pentominoes from the previous activity or have them carefully cut out a set from the Patterns from Pentominoes Activity Sheet.
 - 2) Invite the participants to take their pentominoes and fit them together like jigsaw puzzle pieces. Point out that there are usually several ways to combine pentominoes to fill a given area.



or in several other ways.



- 3) Distribute the Pentomino Puzzles Activity Sheet and have the participants fill in the 3 x 5 and 4 x 5 areas with pentominoes. Demonstrate an example with the pentominoes. Ask the participants to keep a record of their work on a separate piece of grid paper. Have them compare their results with the Solution Sheets.



- 4) Ask the participants how many square units there are in each pentomino. Ask them to find the pentomino with the largest perimeter and the pentomino with the smallest perimeter. Discuss how figures with the same area can have different perimeters.
- 5) Ask participants how many square units there are in all 12 pentominoes combined. Have them design a symmetrical figure with an area equal to 60 square units and then see if it can be completely filled in with all 12 pentominoes. Share results.



Pentomino Puzzles



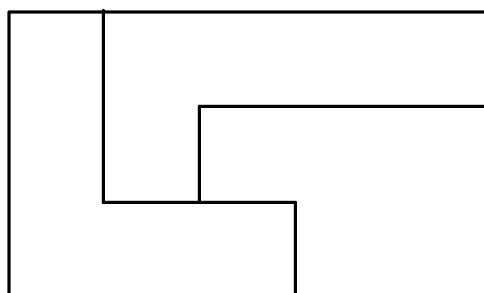
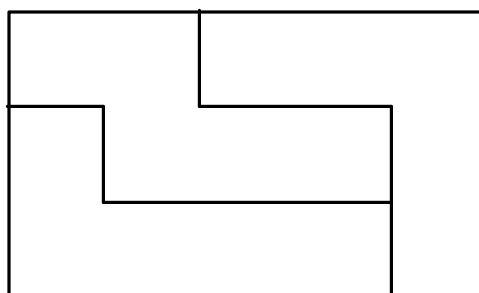
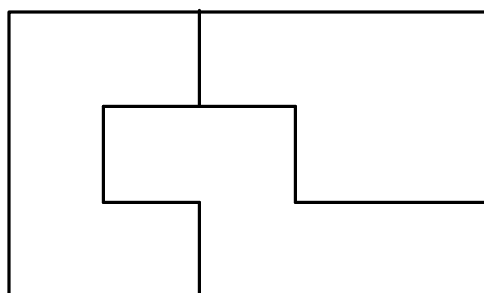
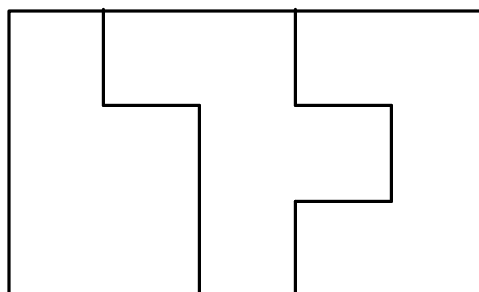
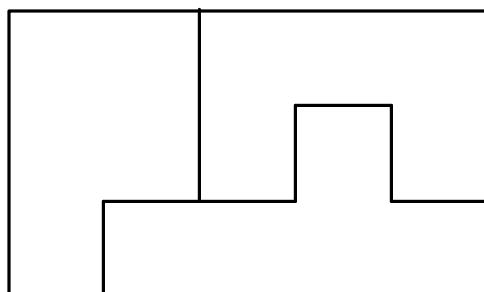
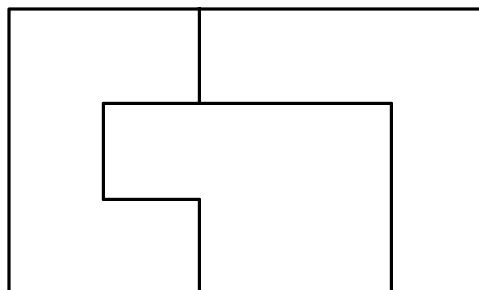
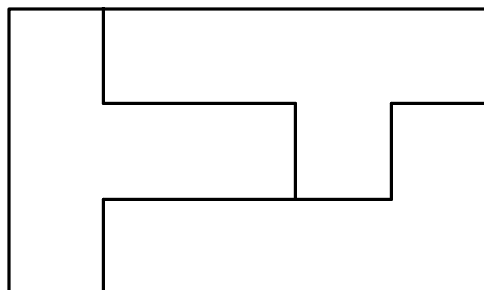
3 x 5



4 x 5

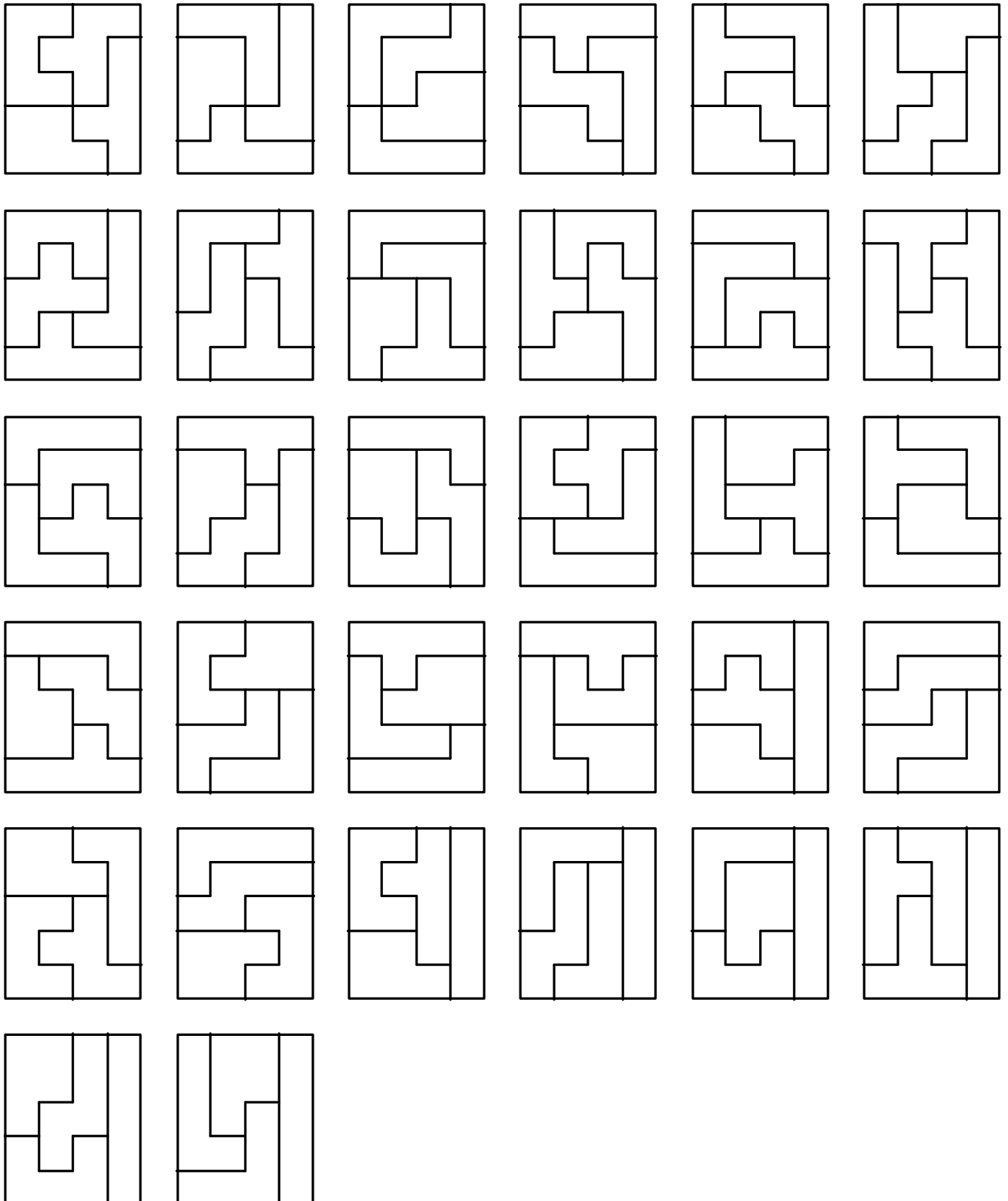


3 x 5 Puzzle Solutions





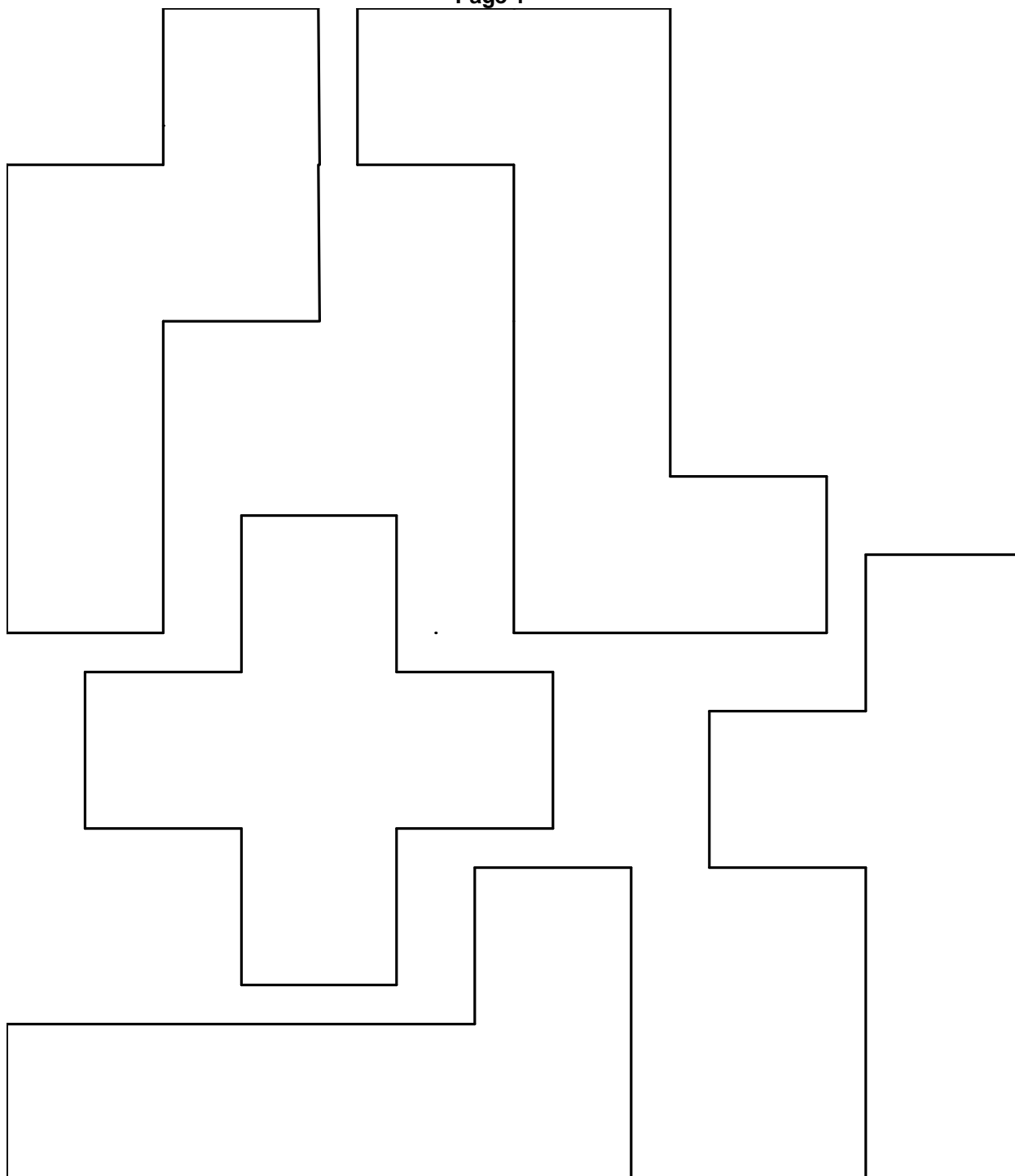
4 x 5 Puzzle Solutions





Patterns for Pentominoes

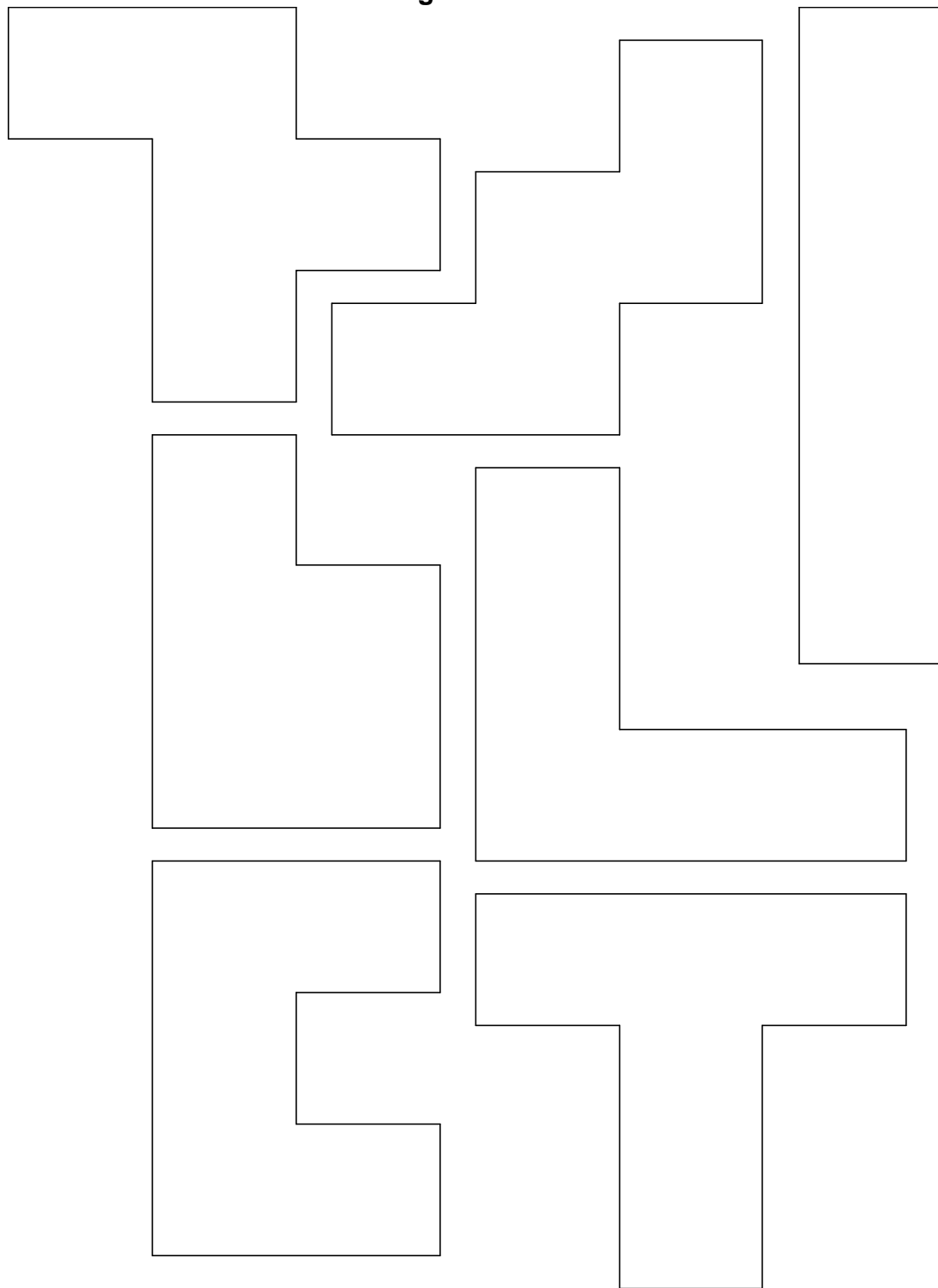
Page 1





Patterns for Pentominoes

Page 2





Activity: Hexominoes

Format: Large Group/Small Group

Objectives: Participants will use spatial visualization to build "hexominoes" with six tiles, testing for figures that are flips (reflections) and turns (rotations) of other figures.

Related SOL: 6.14, 6.15, 8.7, 8.8

Materials: Tiles, scissors, grid paper, Hexominoes Activity Sheets 1, 2, and 3, and envelopes or plastic baggies

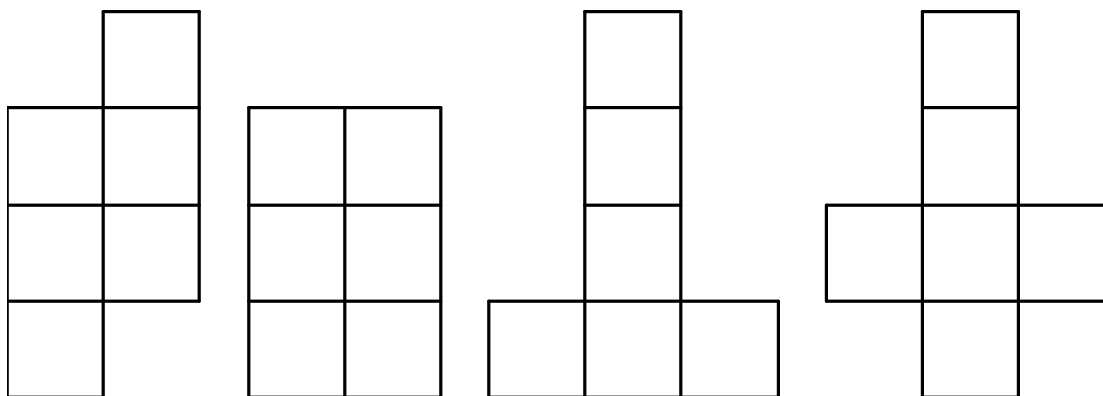
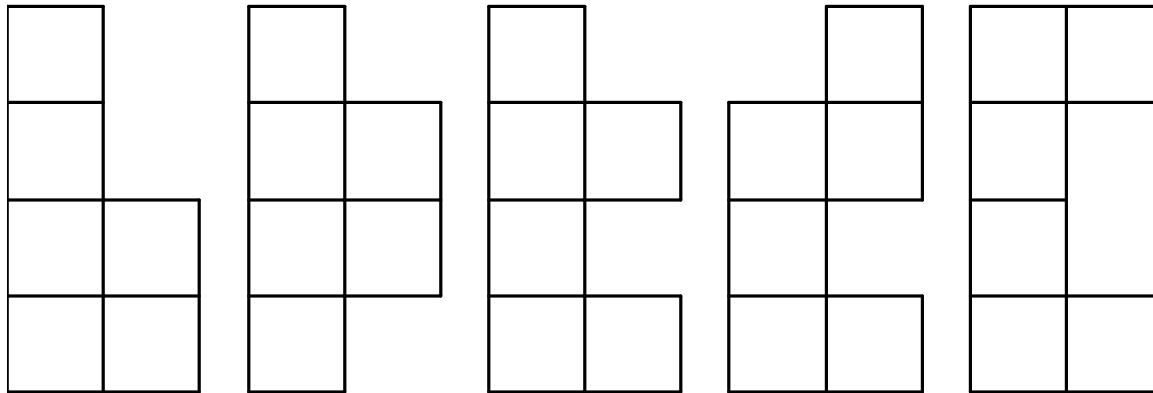
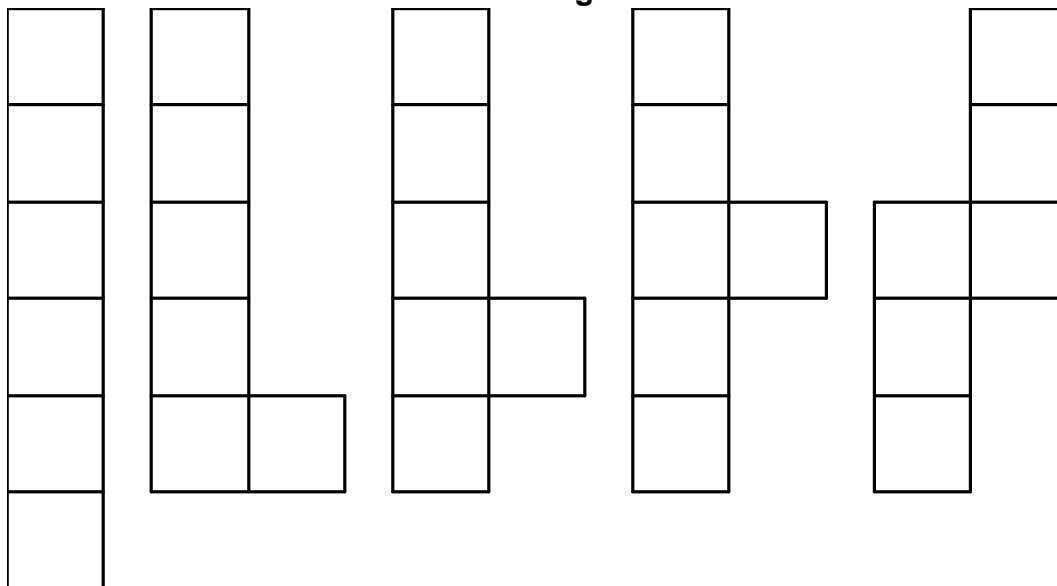
Time Required: Approximately 15 minutes

- Directions:**
- 1) Distribute the materials and ask the participants to:
 - On their own or working with a partner, use six color tiles to find all the possible hexominoes. A **hexomino** is a figure made of six squares, each of which has one side in common with the side of another square.
 - Record each hexomino on grid paper. When you think you have found all the hexominoes, cut them out. Check to make sure that each hexomino is different from all the others.
 - How many hexominoes did you find? (35) Number them on the back and put them in an envelope. Write your names and the number of hexominoes on the envelope.
 - Exchange envelopes with another pair. Check that none of their hexominoes are duplicates of one another. Mark any two that you think are exactly the same. Return the envelopes. Check your envelope to see if any duplicates were found.
 - Decide on a way you can sort your hexominoes.
 - 2) Ask one or two pairs to post their hexominoes in an organized way. Use prompts such as these to promote class discussion:
 - Do you think that you have found all possible hexominoes? Why or why not?
 - In what ways do the hexominoes differ from one another?
 - Did you use a strategy to find new figures? If so, what? Note: one strategy would be to start with each of the pentominoes and place one additional tile in each location around the perimeter of each pentomino to generate all of the hexominoes.
 - Did you find any patterns while making your hexominoes? Did you use these patterns when sorting them? If so, how?
 - Did sorting your hexominoes help you find others that were missing? If so, explain.



Hexominoes

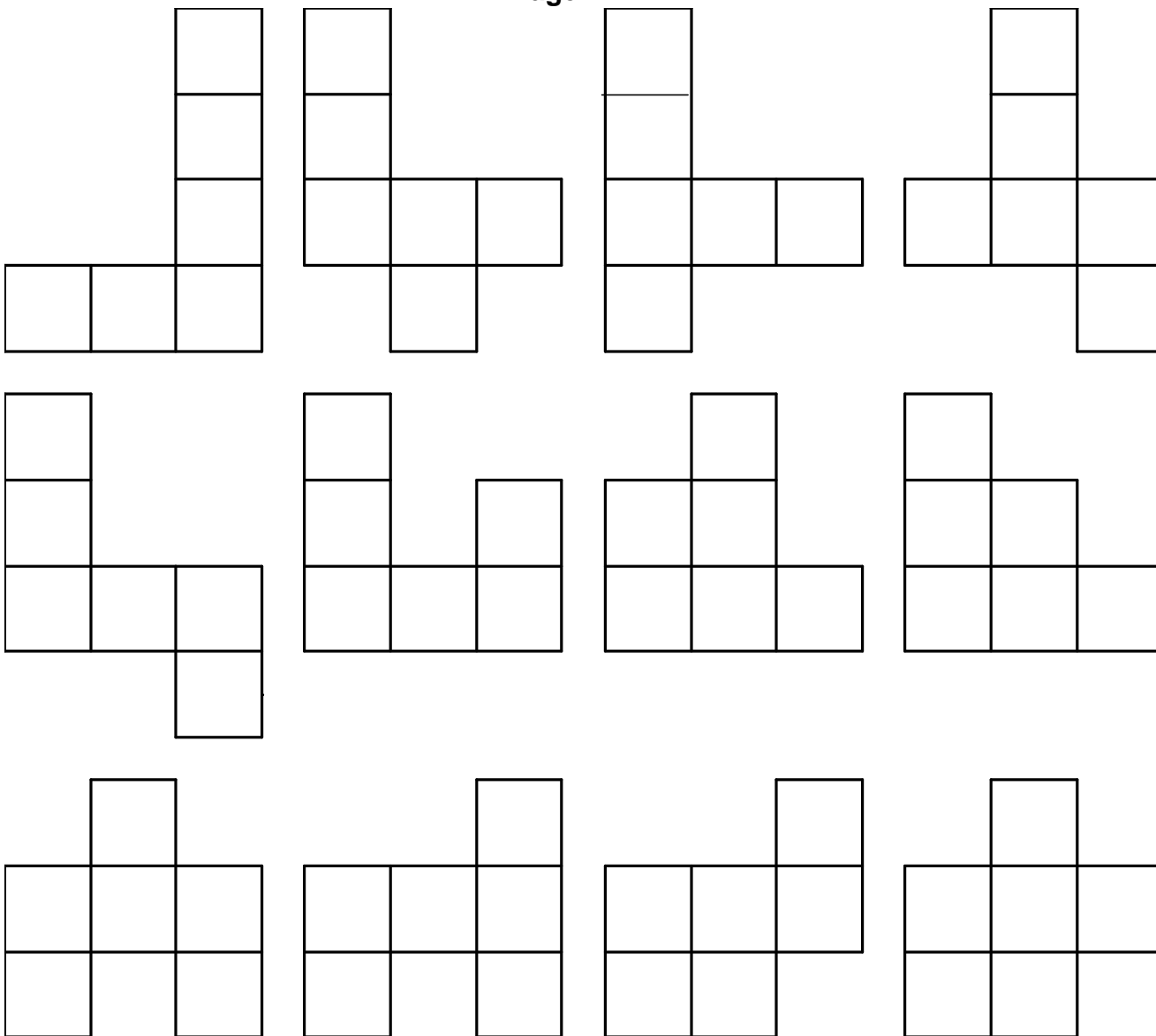
Page 1





Hexominoes

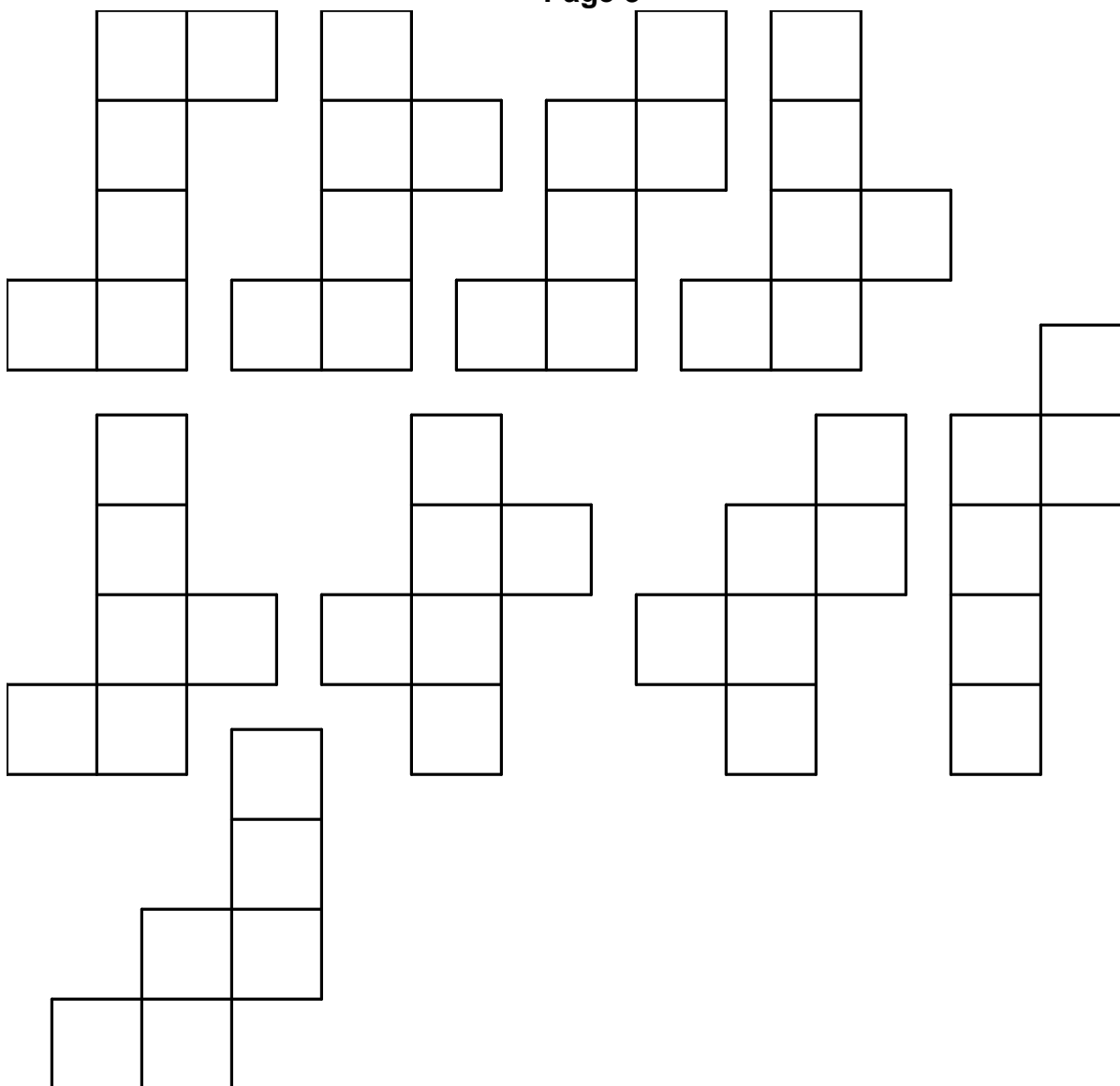
Page 2





Hexominoes

Page 3





Activity: Perimeters with Hexominoes

Format: Large Group/Small Group

Objectives: Participants will sort "hexominoes" according to their perimeters and look for patterns in their collected data.

Related SOL: 6.11, 6.14, 7.7

Materials: Hexominoes from previous activity, Perimeter of Hexominoes Activity Sheet

Time Required: Approximately 15 minutes

Directions: 1) Ask the participants how they would determine the perimeter of each hexomino. Have them sort all of the hexominoes according to their perimeter and then look for patterns in the collected data.

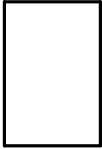
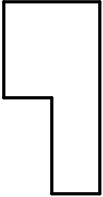
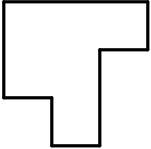
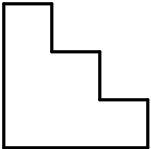
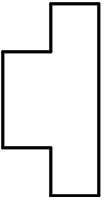
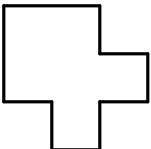
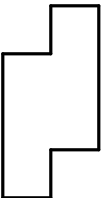
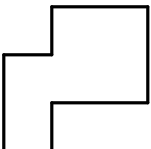
Example Patterns:

- There are no odd numbered perimeters;
- All hexominoes with perimeter 12 include a large square made up of four small squares;
- All hexominoes with perimeter 12 include a rectangle made up of three squares;
- All hexominoes with perimeter 14 include a rectangle made up of two squares; and
- None of the hexominoes with perimeter 14 include a large square made up of four small squares.

2) Ask the participants to determine the area of each hexomino. Discuss how two figures can have the same area but different perimeters.



Perimeters of Hexominoes

10	11	12	13	14
		 		All Others
		 		
		 		
				



Topic: Perimeter and Area, Part II

Description: Participants will explore the relationship between area and perimeter by creating various figures with the same perimeter and different areas, or the same area and different perimeter using both one-inch cubes and geoboards. They will also find the areas of rectangles and triangles by counting squares covered by the figures on a geoboard and then develop formulas for area.

Related SOL: 6.11, 6.14, 7.7, 8.8



Activity: The Perimeter Is 24 Inches. What Is The Area?

Format: Small Group

Objective: Participants will differentiate between perimeter and area, and determine the perimeter and area of various rectangles.

Related SOL: 6.11, 6.14, 7.7, 8.8

Materials: 24-inch paper strip (collar), 38 one-inch cubes, One-Inch Grid Paper, One-Half-Inch Grid Paper

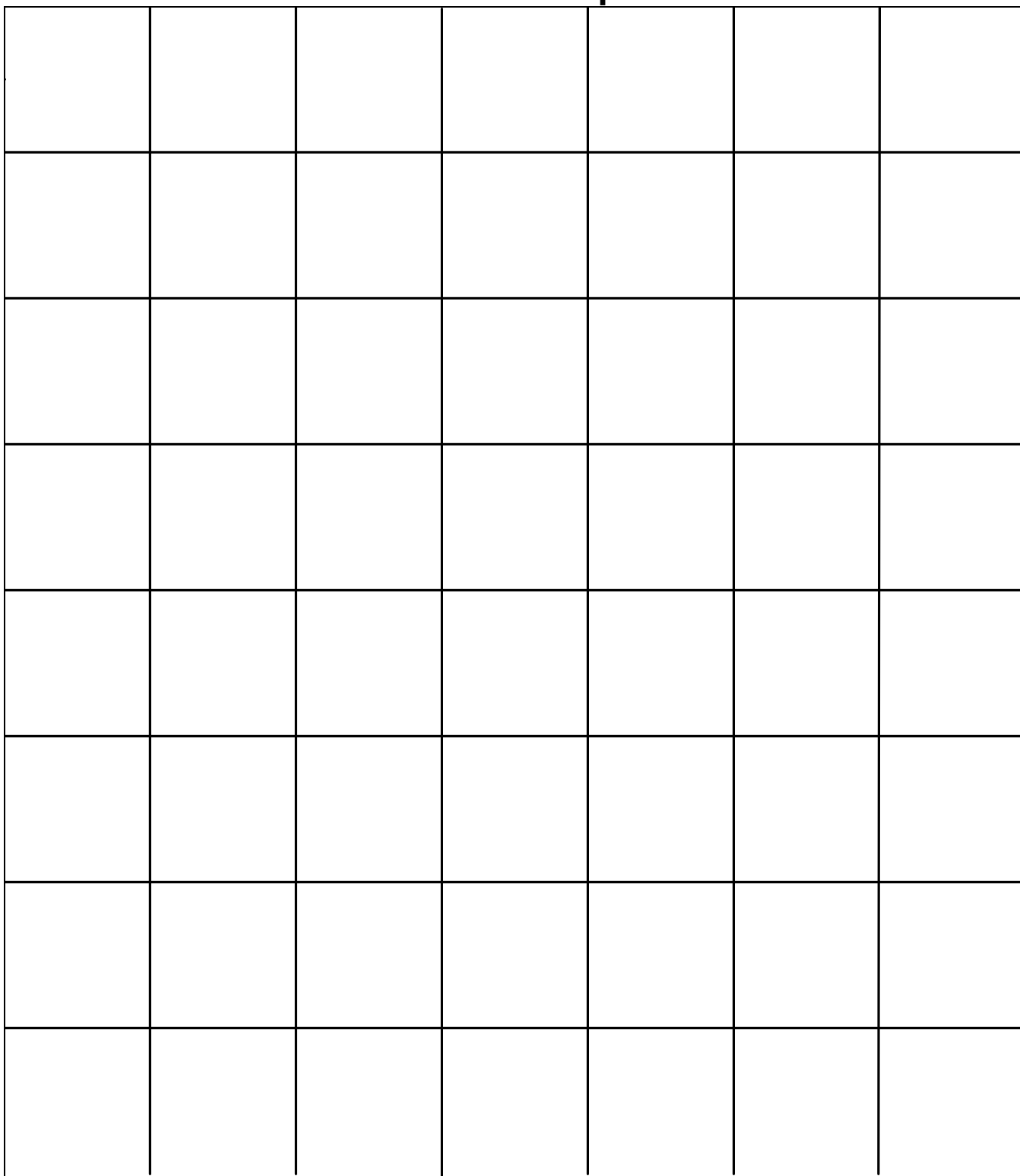
Time Required: Approximately 10 minutes

Directions:

- 1) Organize the participants into teams of two or three.
- 2) Ask the teams to find out how many different rectangular arrays they can make that have a perimeter of 24 inches. Have them make the arrays with the cubes and have them check the perimeter with a 24-inch collar. What is the area of each?
- 3) Once they find a rectangular array, have them draw its representation on the grid paper and write the perimeter and area for each array.

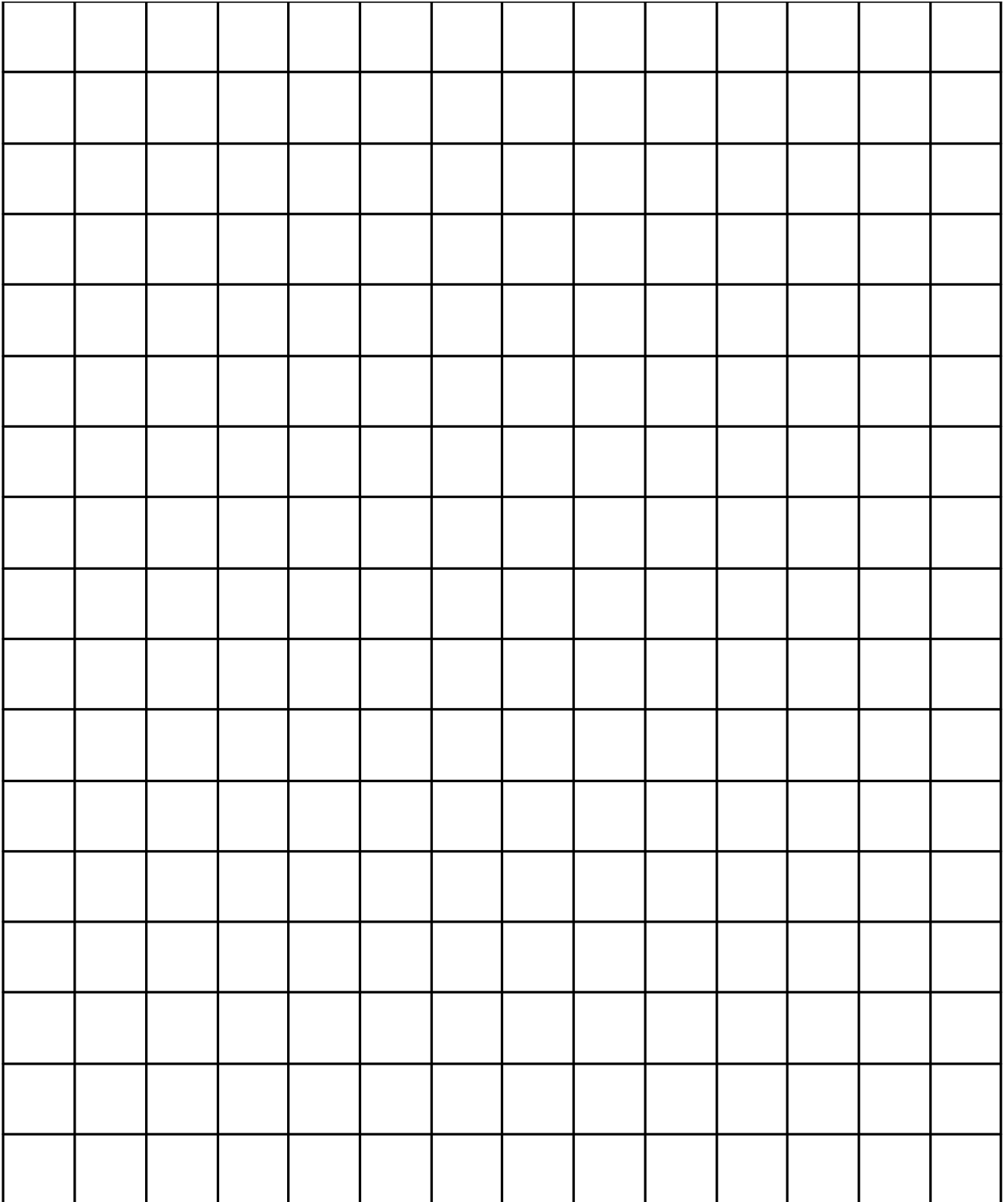


One-Inch Grid Paper





One-Half Inch Grid Paper





Activity: The Area Is 24 Square Inches. What Is The Perimeter?

Format: Small Group

Objective: Participants will differentiate between the perimeter and area, and will determine the perimeter and area of various rectangles.

Related SOL: 6.11, 6.14, 7.7, 8.8

Materials: 24 one-inch cubes, One-Inch Grid Paper, One-Half-Inch Grid Paper

Time required: Approximately 10 minutes

Directions:

- 1) Organize the participants into teams of two or three.
- 2) Ask the participants to find out how many different rectangular arrays they can make that have an area of 24 square inches. What is the perimeter of each?
- 3) Once they find a rectangular array of 24 square inches, have them draw its representation on the grid paper and write the perimeter and area for each array.



Activity: Change The Area

Format: Small Group

Objective: Participants will differentiate between perimeter and area and determine the perimeter and area of various figures without dependence on formulas.

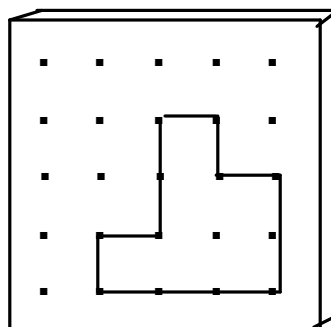
Related SOL: 6.11, 6.14, 7.7

Materials: Square geoboards, rubber bands, Geoboard Dot Paper

Time required: Approximately 10 minutes

Directions:

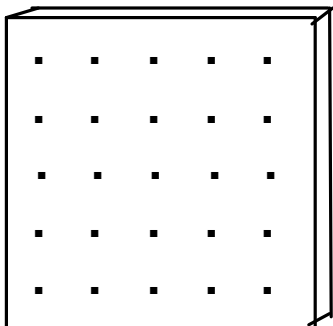
- 1) Organize the participants into teams of two or three.
- 2) Have them copy this figure on the geoboard and onto dot paper, labeling its area and perimeter.



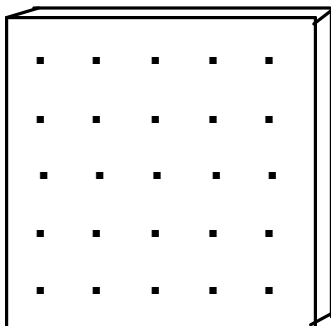
- 3) Have the participants change the figure to make another figure that has the same area and a larger perimeter, recording it on dot paper with its area and perimeter.
- 4) Have the participants change the figure to make another figure that has the same area and a smaller perimeter, recording it on dot paper with its area and perimeter.
- 5) Have the participants make three more figures that have different perimeters but the same area, recording them on dot paper. Discuss the results.



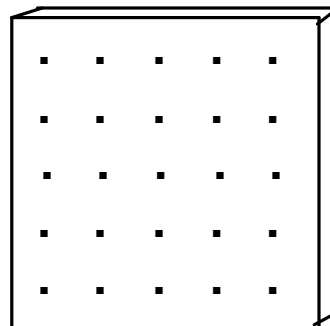
Geoboard Dot Paper



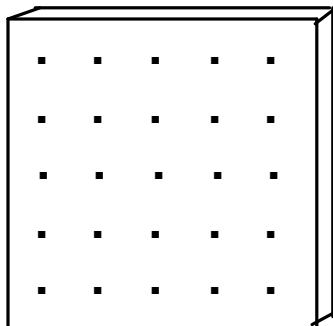
Perimeter: _____
Area: _____



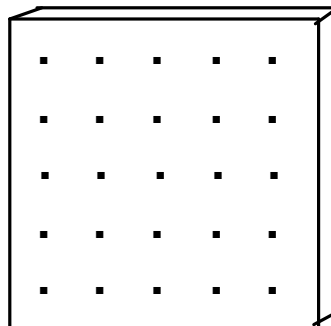
Perimeter: _____
Area: _____



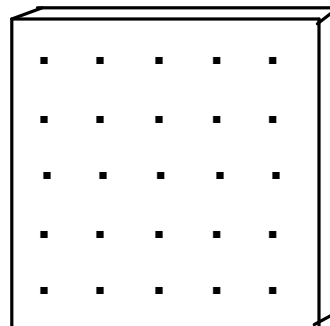
Perimeter: _____
Area: _____



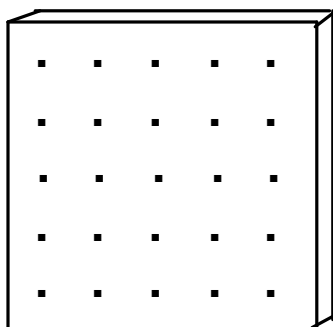
Perimeter: _____
Area: _____



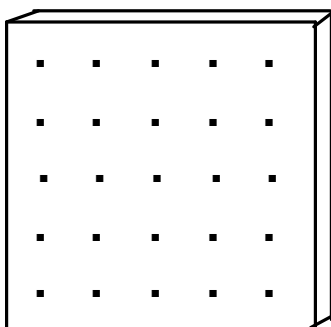
Perimeter: _____
Area: _____



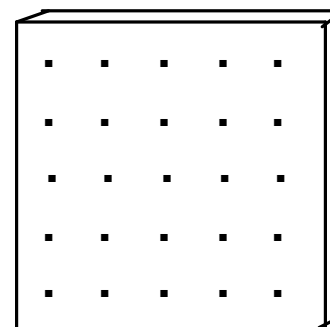
Perimeter: _____
Area: _____



Perimeter: _____
Area: _____



Perimeter: _____
Area: _____



Perimeter: _____
Area: _____